4.3 Binomial, normal and Poisson models

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS expected value

Objective

Binomial, normal and Poisson models. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** : The probability-weighted mean of a random variable.

The method

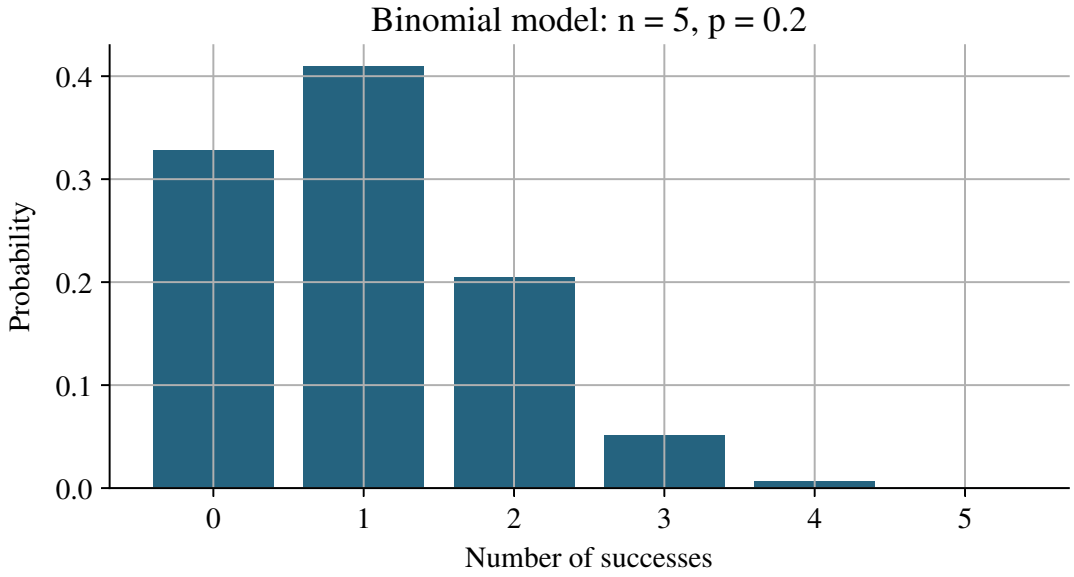
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- E(X)=np and Var(X)=np(1-p).
- For a normal model use z=(x-μ)/σ.
- A Poisson model describes counts with a constant rate and appropriate independence assumptions.

A worked case

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}, \qquad X\sim B(n,p)$$

Example:

For X binomial(5,0.2), P(X=0)=0.8^5=0.32768, E(X)=1 and Var(X)=0.8. For a normal quantity with μ=100,σ=15, the value 130 has z=2. A Poisson mean of 3 per hour gives mean 6 over two hours.



Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

Not true: Every count of successes has a binomial distribution regardless of dependence. The binomial model needs independent trials with a fixed probability; dependent trials break it.

Applying it

- Write the event as an inequality before using calculator distribution functions.
- Distinguish $P(X<k)$, $P(X\leq k)$ and a tail complement.
- State assumptions in context.

In your course

- Applications and Interpretation SL, the course first assessed in 2021.

Key:

- **Expected value**: The probability-weighted mean of a random variable.
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- Watch for: not every count is binomial: changing p or dependence can invalidate it.

■ **Write from a z-score of 4**

- A value 90 comes from a distribution with mean 70 and standard deviation 5.
- Find the z-score. A binomial count needs independent trials.

2 Practice

2.1 For X binomial(5,0.2), find P(X=0). Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.2 Find Var(X) for X binomial(5,0.2). Show the relation, substitution and units. [2]

2.3 For μ=100 and σ=15, find the z-score of 130. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Every count of successes has a binomial distribution regardless of dependence. [2]

2.5 A count of successes fails the binomial model when which condition breaks? Choose one answer.

- **A** Each trial has exactly two outcomes, which rules out the binomial model
- **B** The number of trials is fixed in advance, so the count cannot be binomial
- **C** The success probability changes between trials or trials are dependent [1]

3 Exam-style questions

3.1 For X~Binomial(5, 0.2), find P(X≥1) and the mean. Explain one situation in which the binomial model would be inappropriate. [4]

Solutions

2.1 No successes means five failures: 0.8^5=0.32768.

2.2 Binomial variance=np(1-p)=5×0.2×0.8=0.8.

2.3 Standardise: z=(130-100)/15=2.

2.4 False. The binomial model needs independent trials with a fixed probability; dependent trials break it.

2.5 C The binomial model needs fixed n, independent trials, two outcomes and a constant p. Sampling without replacement makes trials dependent, so the binomial formula no longer applies exactly.

3.1 $1-0.8^5$=0.67232; mean np=1. Trials must be independent with constant success probability and fixed count; depletion or correlated outcomes can violate this.