2.4 Cubic, sinusoidal and piecewise models

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS cubic • sinusoidal • amplitude • period • principal axis • piecewise

Objective

Build the skills to answer exam questions on **cubic, sinusoidal and piecewise models** 、 , and on the whole **modelling cycle** .

You must be able to:

- use cubic models $f(x) = ax^3 + bx^2 + cx + d$ and find maxima with the GDC
- use sinusoidal models $f(x) = a\sin(bx) + d$ and $f(x) = a\cos(bx) + d$ (degrees): amplitude, period $\dfrac{360°}{b}$, principal axis
- use piecewise models with different rules on different intervals
- choose, fit, test and criticize a model

1 Key knowledge and worked examples

■ Cubic, sinusoidal and piecewise models

Sinusoidal $f(x) = a\sin(bx) + d$: amplitude $|a| = \dfrac{\max - \min}{2}$; principal axis $y = d = \dfrac{\max + \min}{2}$; period $= \dfrac{360°}{b}$.

pattern in the data	model
repeats (tides, temperature over a year)	sinusoidal
rises, falls, rises (or one turning point and an inflection)	cubic
different rules for different ranges (tax bands, phone plans)	piecewise

Modelling cycle: choose a model from the shape → find the parameters (regression or simultaneous equations) → test it on data you did not use → state its limits.

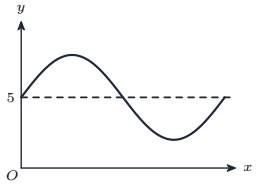
Worked example. High tide 9 m, low tide 3 m: $d = \mathbf{6}$ and $a = \mathbf{3}$.

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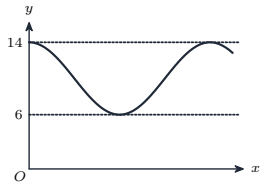
Answer shapes: for a time in a sinusoidal model, give every solution in the interval asked for.

2 Practice

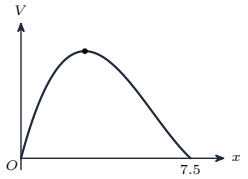
2.1 $y = 3\sin(30x) + 5$. Write down the amplitude and the equation of the principal axis. [2]



2.2 For $y = 4\cos(45x) + 10$, find the period, the maximum value and the minimum value. [3]



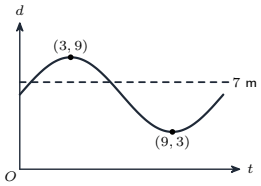
2.3 An open box is made from a 20 cm by 15 cm sheet by cutting squares of side x cm from the corners, so $V = x(20 - 2x)(15 - 2x)$. Use your GDC to find the greatest volume. [3]



3 Exam-style questions

3.1 The depth of water in a harbour is $d(t) = a\sin(bt) + c$ metres, t hours after midnight. High tide is 9 m at $t = 3$; the next low tide is 3 m at $t = 9$.

(a) Find a, b and c. [4]



(b) Find the depth at $t = 5$. [2]

(c) A ship needs a depth of at least 7 m. Find how long it can stay in the harbour between $t = 0$ and $t = 12$. [3]

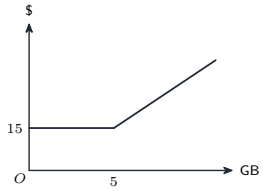
3.2 A phone plan costs \$15 a month for up to 5 GB of data, plus \$4 for each GB above 5 GB.

(a) Find the monthly cost for 3 GB and for 8 GB. [2]

(b) A bill is \$35. Find the data used. [2]

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(c) A different quantity y is measured: $(1, 2)$, $(2, 3)$, $(3, 6)$, $(4, 17)$. Find the cubic model that fits these points. [3]



(d) Use the model to predict y when $x = 5$, and comment on how reliable this is. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

amplitude **3**; $\mathbf{y = 5}$

2.2

period $\frac{360}{45} = \mathbf{8}$; max **14**; min **6**

2.3

graph for $0 < x < 7.5$; $x = 2.83$; $V = \mathbf{379}$ cm^3

3.1

(a) $c = \mathbf{6}$; $a = \mathbf{3}$; period 12, $b = \mathbf{30}$

(b) $6 + 3\sin 150° = \mathbf{7.5}$ m

(c) $3\sin(30t) + 6 = 7$; $t = 0.649$ and 5.351; **4.70** hours

3.2

(a) \$**15**; $15 + 4 \times 3 = \$\mathbf{27}$

(b) $15 + 4(x - 5) = 35$; $x = \mathbf{10}$ GB

(c) cubic regression; $\mathbf{y = x^3 - 5x^2 + 9x - 3}$

(d) **42**; $x = 5$ is outside the data, and a cubic through four points fits them exactly, so the prediction may be poor (R1)

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