

Exercise walk-through

4.2

Probability, trees and conditional reasoning

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation

4.2 Probability, trees and conditional reasoning

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS conditional probability

Objective

Probability, trees and conditional reasoning. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

■ What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event

after restricting the sample space to a stated condition.

■ The method

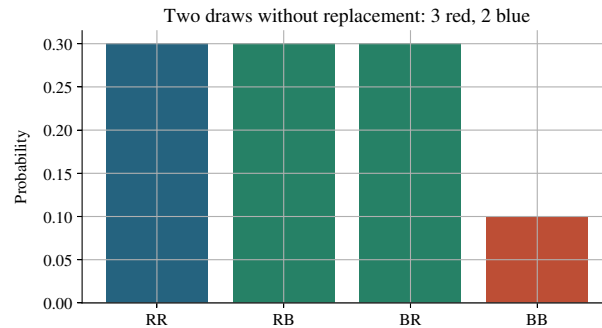
- Multiply along a tree branch and add disjoint branches.
 - With replacement, the composition stays fixed.
 - Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.
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■ A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Example:

Without replacement, $P(\text{two red}) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$. $P(\text{one of each}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$.
If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



■ Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

Not true: Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B) = 0 \neq P(A)P(B)$.

■ Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

■ In your course

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Conditional probability:** The probability of an event after restricting the sample space to a stated condition.
 - Multiply along a tree branch and add disjoint branches.
 - Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
-

■ Write from a conditional probability of 0.70

- $P(A \text{ and } B)$ is 0.28 and $P(B)$ is 0.40.
- Find $P(A \text{ given } B)$. Mutually exclusive events that can happen are not independent.

Exercises 2.1-2.3

2.1 Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

2.2 Find $P(A \text{ given } B)$ if $P(A \cap B) = 0.12$ and $P(B) = 0.3$. Show the relation, substitution and units. [2]

2.3 Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

2.1 Without replacement, $P(RR)=3/5 \times 2/4=0.3$.

2.2 Find $P(A \text{ given } B)$ if $P(A \cap B)=0.12$ and $P(B)=0.3$. Show the relation, substitution and units. [2]

2.2 $P(A|B)=P(A \cap B)/P(B)=0.12/0.3=0.4$.

2.3 Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

2.3 Add the two orders: $3/5 \times 2/4 + 2/5 \times 3/4 = 0.6$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Conditional probability means the probability of an event after restricting the sample space to a stated condition. [2]

2.5 Two disjoint events A and B both have positive probability. Can they be independent? Choose one answer.

- **A** Yes: disjoint and independent mean the same thing
- **B** No: if A happens, $P(B)$ drops to 0, so knowing A changes B
- **C** Yes: independence is automatic for all events

[1]

Exercises 2.4-2.5

2.4 Decide whether this claim is true and justify: Conditional probability means the probability of an event after restricting the sample space to a stated condition. [2]

2.4 True. True: The probability of an event after restricting the sample space to a stated condition.

2.5 Two disjoint events A and B both have positive probability. Can they be independent? Choose one answer.

- **A** Yes: disjoint and independent mean the same thing
- **B** No: if A happens, $P(B)$ drops to 0, so knowing A changes B
- **C** Yes: independence is automatic for all events

[1]

2.5 B Disjoint means A and B cannot happen together, so $P(B \text{ given } A) = 0 \neq P(B)$. Independence requires that knowing one does not change the other - disjoint events with positive probability always fail that.

Exercise 3.1

3.1 A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

Exercise 3.1

3.1 A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

3.1 $P(RR) = \frac{3}{5} \times \frac{2}{4} = 0.3$. $P(\text{at least one } R) = 1 - \frac{2}{5} \times \frac{1}{4} = 0.9$. Conditional probability $= 0.3 / 0.9 = 1/3$. The conditioning changes the sample space.