

Exercise walk-through

2.2

Exponentials, logarithms and modelling

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation

2.2 Regression, financial models and residuals

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS residual

Objective

Regression, financial models and residuals. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

■ Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
- This lesson studies **residual** : The observed value minus the value predicted by

a fitted model.

■ The method

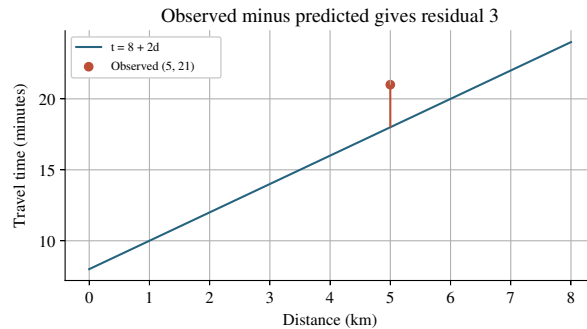
- A linear model $y=a+bx$ has intercept a and slope b with contextual units.
 - Inspect residuals and the data range.
 - For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.
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■ A worked case

$$\hat{t} = 8 + 2d, \quad e = t - \hat{t}$$

Example:

The fitted model $t=8+2d$ predicts 18 minutes at $d=5$ km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes $1000 \times 1.05^3 = 1157.625$ after 3 years.



■ **Where it goes wrong**

- A good fit does not prove a causal mechanism.
- A correlation coefficient measures linear association, not the gradient.
- Calculator output must be translated into a model, checked and interpreted.

Not true: A high correlation permits reliable extrapolation to any distance. A model fitted to data is reliable only near that data; extrapolating far outside it is unreliable.

■ **Applying it**

- In IB AI, record the data source, domain, assumptions and calculator method.
- Compare an alternative model and judge predictions against residuals.
- A financial answer must state payment timing and whether interest is compounded.

■ **In your course**

- Applications and Interpretation SL, the course first assessed in 2021.

Key:

- **Residual:** The observed value minus the value predicted by a fitted model.
 - A linear model $y=a+bx$ has intercept a and slope b with contextual units.
 - Watch for: A good fit does not prove a causal mechanism.
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■ Write from a prediction of 17

- The model $t = 5 + 3d$ predicts the time at $d = 4$.
- An observed time of 25 leaves a residual. A high correlation does not license extrapolation.

Exercises 2.1·2.3

2.1 For $t=8+2d$, find t at $d=5$. Show the relation, substitution and units. [2]

2.2 Find the residual if observed $t=21$ and predicted $t=18$. Show the relation, substitution and units. [2]

2.3 Find the value of 1000 after 3 years at 5% compound annual interest. Give it to 3 significant figures. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For $t=8+2d$, find t at $d=5$. Show the relation, substitution and units. [2]

2.1 Substitute distance 5: $8+2\times 5=18$ minutes.

2.2 Find the residual if observed $t=21$ and predicted $t=18$. Show the relation, substitution and units. [2]

2.2 Residual=observation-prediction= $21-18=3$ minutes.

2.3 Find the value of 1000 after 3 years at 5% compound annual interest. Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.3 Compound annually: $1000\times 1.05^3=1157.625$.

Exercises 2.4-2.5

2.4 Decide whether this claim is true and justify: A high correlation permits reliable extrapolation to any distance. [2]

2.5 The model $t=8+2d$ fits observed journey times. An observed 21 minutes against a predicted 18 means what? Choose one answer.

- **A** A residual of +3 minutes: the model under-predicted this observation
- **B** The model is exact, so the 3-minute gap must be a measurement error
- **C** The observation should be discarded because it does not fit the model [1]

Exercises 2.4-2.5

2.4 Decide whether this claim is true and justify: A high correlation permits reliable extrapolation to any distance. [2]

2.4 False. A model fitted to data is reliable only near that data; extrapolating far outside it is unreliable.

2.5 The model $t=8+2d$ fits observed journey times. An observed 21 minutes against a predicted 18 means what? Choose one answer.

- **A** A residual of +3 minutes: the model under-predicted this observation
- **B** The model is exact, so the 3-minute gap must be a measurement error
- **C** The observation should be discarded because it does not fit the model [1]

2.5 A Residual = observed - predicted = $21-18 = 3$. Residuals reveal systematic model weakness; a good fit on average can still hide poor regions.

Exercise 3.1

3.1 A fictional journey model is $t=8+2d$. An observed trip at $d=5$ takes 21 minutes. Find the residual, predict at $d=50$ and explain why that prediction needs caution. [4]

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3.1 Prediction 18, residual +3 minutes; at 50 prediction 108. If the fitted distance range was short, this is extrapolation; traffic, stops and model form may change. Regression does not establish a causal coefficient by itself.