

Exercise walk-through

# 1.3

## Sequences, series and recurrence

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INTERNATIONAL BACCALAUREATE · IB DIPLOMA ·  
MATHEMATICS: APPLICATIONS AND INTERPRETATION · SL · EXERCISE  
SHEET

# 1.3 Sequences, series and recurrence

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 13 marks**

**KEY WORDS** common ratio

# Objective

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Sequences, series and recurrence. Original classroom practice for IB SL; not an official IB paper.

## 1 Key knowledge and worked examples

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### ■ Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** : The constant multiplier between consecutive

terms of a geometric sequence.

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## ■ The method

- For an arithmetic progression,  $u_n = a + (n-1)d$  and  $S_n = n(2a + (n-1)d)/2$ .
  - For a geometric progression,  $u_n = ar^{n-1}$  and  $S_n = a(1-r^n)/(1-r)$ .
  - An infinite geometric sum exists only if  $|r| < 1$ .
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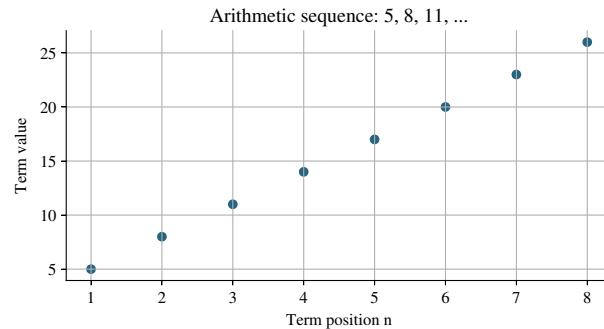
## ■ A worked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

### Example:

For  $a=5, d=3, n=8$ ,  $u_8=5+7 \times 3=26$  and  $S_8=8(10+21)/2=124$ . For  $a=12, r=1/2$ ,  $S_{\infty}=12/(1-1/2)=24$ . For  $u_{(n+1)}=2u_n+1$  with  $u_1=1$ , the next terms are 3, 7, 15.

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■ **Where it goes wrong**

- The first term has index 1, so the exponent is  $n-1$ .
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

**Not true:** Every geometric series has a finite sum to infinity. A sum to infinity exists only when  $-1 < r < 1$ ; for  $r=2$  the terms grow without limit.

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■ **Applying it**

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

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■ **In your course**

- Applications and Interpretation SL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

**Key:**

- **Common ratio:** The constant multiplier between consecutive terms of a geometric sequence.
  - For an arithmetic progression,  $u_n = a + (n-1)d$  and  $S_n = n(2a + (n-1)d)/2$ .
  - Watch for: the first term has index 1, so the exponent is  $n-1$ .
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■ **Write from the 5th term 23**

- An arithmetic sequence has first term 7 and common difference 4.
- Find the 5th term. An infinite geometric series needs a common ratio with size below 1.

## Exercises 2.1-2.3

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**2.1** For  $a=5, d=3$ , find  $u_8$ . Show the relation, substitution and units. [2]

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**2.2** Find the sum of the first 8 terms with  $a=5, d=3$ . Show the relation, substitution and units. [2]

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**2.3** Find the infinite sum with  $a=12, r=0.5$ . Show the relation, substitution and units. [2]

## Exercises 2.1-2.3

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**2.1** For  $a=5, d=3$ , find  $u_8$ . Show the relation, substitution and units. [2]

**2.1**  $u_8 = 5 + (8-1) \times 3 = 26.$

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**2.2** Find the sum of the first 8 terms with  $a=5, d=3$ . Show the relation, substitution and units. [2]

**2.2**  $S_8 = 8[2 \times 5 + 7 \times 3] / 2 = 124.$

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**2.3** Find the infinite sum with  $a=12, r=0.5$ . Show the relation, substitution and units. [2]

**2.3** Since  $|r| < 1$ ,  $S_\infty = 12 / (1 - 0.5) = 24.$

## Exercises 2.4-2.5

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**2.4** Decide whether this claim is true and justify: Common ratio means the constant multiplier between consecutive terms of a geometric sequence. [2]

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**2.5** An infinite geometric series has a finite sum only when which condition holds? Choose one answer.

- **A** The common ratio is a whole number
- **B** The first term is positive
- **C** The common ratio satisfies  $|r| < 1$

[1]

## Exercises 2.4-2.5

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**2.4** Decide whether this claim is true and justify: Common ratio means the constant multiplier between consecutive terms of a geometric sequence. [2]

**2.4** True. True: The constant multiplier between consecutive terms of a geometric sequence.

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**2.5** An infinite geometric series has a finite sum only when which condition holds? Choose one answer.

- **A** The common ratio is a whole number
- **B** The first term is positive
- **C** The common ratio satisfies  $|r| < 1$

[1]

**2.5 C** The terms  $ar^{n-1}$  shrink towards zero only when  $|r| < 1$ , so the sum converges to  $a/(1-r)$ . With  $|r| \geq 1$  the terms do not shrink and the sum diverges.

## Exercise 3.1

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**3.1** A sequence obeys  $u_1=4$  and  $u_{n+1}=0.5 u_n+3$ . Find its fixed point and express  $u_n-6$  as a geometric sequence. Does the sum of  $u_n$  converge? [4]

## Exercise 3.1

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**3.1** Fixed point 6. Subtracting 6 gives  $u_{n+1}-6=0.5(u_n-6)$ , so  $u_n=6-2(0.5)^{n-1}$ . Terms approach 6, so their infinite sum diverges; a convergent sequence need not give a convergent series.