

Exercise walk-through

1.1

Exact arithmetic and estimation

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Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS prime factor

Objective

Exact arithmetic and estimation. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

■ Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer

exactly.

■ The method

- Prime factors reveal shared structure.
 - Use the smallest common prime powers for the HCF and the largest for the LCM.
 - Estimate before calculating; use brackets to preserve the order of operations.
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■ A worked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

Example:

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{Shared smallest powers: } 2 \times 3^2 = 18$$

■ **Where it goes wrong**

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

Not true: The HCF of two positive integers is always larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

■ **Applying it**

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

■ **In your course**

- Applications and Interpretation SL, the course first assessed in 2021.

Key:

- **Prime factor:** A prime number that divides the integer exactly.
 - Prime factors reveal shared structure.
 - Watch for: the HCF divides both numbers; the LCM is a multiple of both.
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■ Write from an HCF of 14

- Find the HCF and LCM of 28 and 42.
- The HCF is smaller than both integers.

Exercises 2.1-2.3

2.1 Find the HCF of 48 and 60. Show the relation, substitution and units. [2]

2.2 Find the LCM of 8 and 12. Show the relation, substitution and units. [2]

2.3 Work out $\frac{3}{4} + \frac{5}{6}$ as a decimal. Give it to 3 significant figures. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find the HCF of 48 and 60. Show the relation, substitution and units. [2]

2.1 $48=2^4 \times 3$ and $60=2^2 \times 3 \times 5$. The shared smallest powers give $\text{HCF}=2^2 \times 3=12$.

2.2 Find the LCM of 8 and 12. Show the relation, substitution and units. [2]

2.2 $8=2^3$ and $12=2^2 \times 3$. The largest prime powers give $\text{LCM}=2^3 \times 3=24$.

2.3 Work out $3/4 + 5/6$ as a decimal. Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.3 Use denominator 12: $3/4+5/6=9/12+10/12=19/12$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: The HCF of two positive integers is always larger than either integer. [2]

2.5 Which statement correctly relates HCF and LCM of two positive integers? Choose one answer.

- **A** The HCF is always the larger of the two numbers
- **B** The HCF divides both; the LCM is a multiple of both
- **C** The LCM always equals their product

[1]

Exercises 2.4-2.5

2.4 Decide whether this claim is true and justify: The HCF of two positive integers is always larger than either integer. [2]

2.4 False. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

2.5 Which statement correctly relates HCF and LCM of two positive integers? Choose one answer.

- **A** The HCF is always the larger of the two numbers
- **B** The HCF divides both; the LCM is a multiple of both
- **C** The LCM always equals their product

[1]

2.5 B The highest common factor must divide each number, and the lowest common multiple must be a multiple of each. Only when the numbers share no factor does the LCM equal the product.

Exercise 3.1

3.1 A supplier has 84 pencils and 126 pens. Find the greatest number of identical bags without leftovers and the contents of each; then explain why the LCM answers a different question. [4]

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3.1 $\text{HCF}=42$, each bag 2 pencils and 3 pens. $\text{LCM}=252$ is a common multiple, useful for matching repeated quantities, not the greatest number of bags dividing both stocks.