Functions

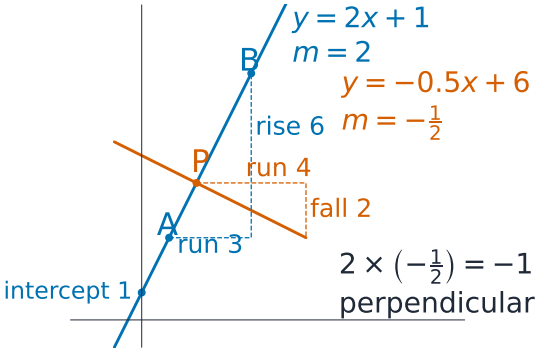
Topic 2 · SL and HL

A law that held for fifty years

> A model is a function chosen to describe data and predict.

- Moore, 1965: chip components double regularly.
- Revised in 1975 to every two years.
- This unit: lines $\rightarrow$ functions $\rightarrow$ models $\rightarrow$ testing.

Gradients



- $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- **Perpendicular** gradients multiply to -1.

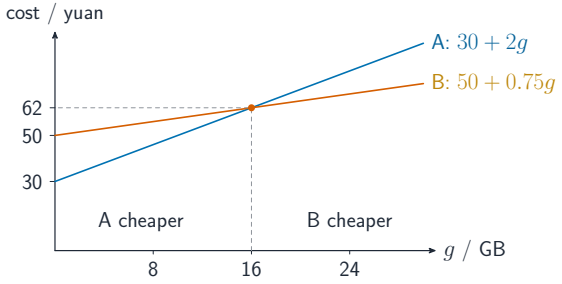
Linear models

1. **Plan A.** $30 + 2g$
2. **Plan B.** $50 + 0.75g$
3. **Equal.** $1.25g = 20$, so $g = 16$ GB

Decision

Interpret both parameters in context

When do two plans cost the same?

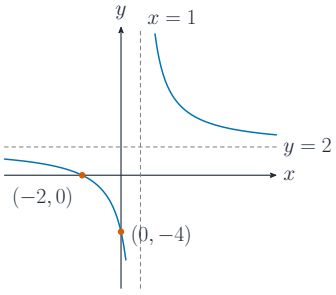


A is cheaper below, B above

Features with technology

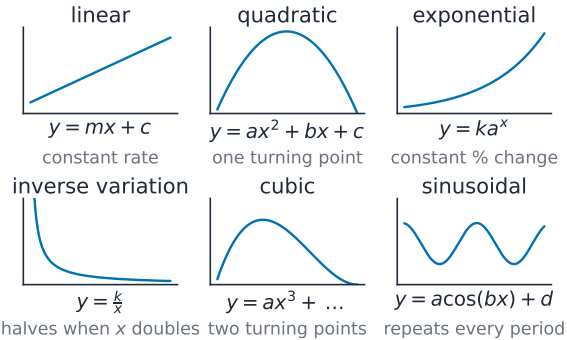
Feature	$f(x) = \frac{6}{x-1} + 2$
Vertical **asymptote**	$x = 1$
Horizontal asymptote	$y = 2$
y-intercept	$(0, -4)$
Zero	$(-2, 0)$

Features of a rational function



Find the asymptotes first

Model shapes



- Choose by how the quantity changes.
- Then fit and test.

A thrown ball

1. **Axis.** $t = -\frac{14}{2(-4.9)} \approx 1.43$ s
2. **Maximum.** $h \approx 11.5$ m
3. **Lands.** $h = 0$ at $t \approx 2.96$ s

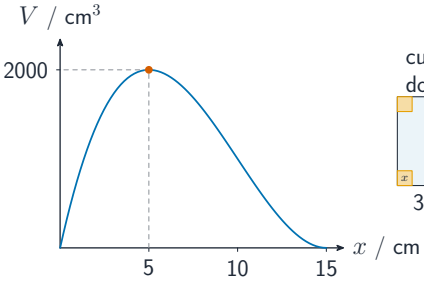
Decision

State the **domain** of the model

Exponential and sinusoidal

Model	Result
$A = 200 \times 0.85^{t}$	below 50 mg after 8.53 h
$I = \frac{48}{d^2}$	$I = 3$ at $d = 4$
$h = 1.8\cos(30t^\circ) + 4.2$	2.4 m to 6 m, **period** 12 h

The largest open box



cut squares of side x;
domain $0 < x < 15$



The **domain** is $0 < x < 15$

Check: which model?

Your turn

Which model fits these values best?

A. $y = 5 \times 1.5^x$

B. $y = 5 + 2.5x$

C. $y = 5x^2$

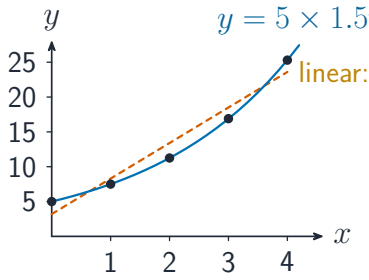
D. $y = \frac{5}{x}$

Is the model reasonable?

> 3000×1.04^{100} is over 150 000; no data support that.

- **Interpolation** is safer than **extrapolation**.
- No pattern in the errors.

Which model fits?



linear: misses the bend

differences 2.5, 3.75, 5.6, 8.4 grow
ratios all about 1.5: exponential

Choose the model from the shape and the context

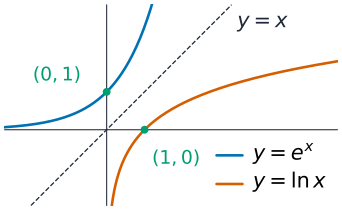
Composite and inverse

1. **Write.** $y = \frac{x+4}{3}$
2. **Swap.** $x = \frac{y+4}{3}$
3. **Rearrange.** $f^{-1}(x) = 3x - 4$

Decision

Check $f(f^{-1}(x)) = x$

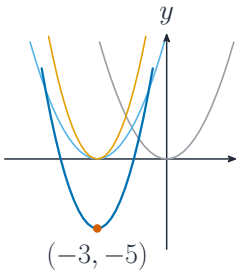
Inverse functions reflect in $y = x$



ln and exp inverse · reflect in $y = x$

Domain and **range** swap

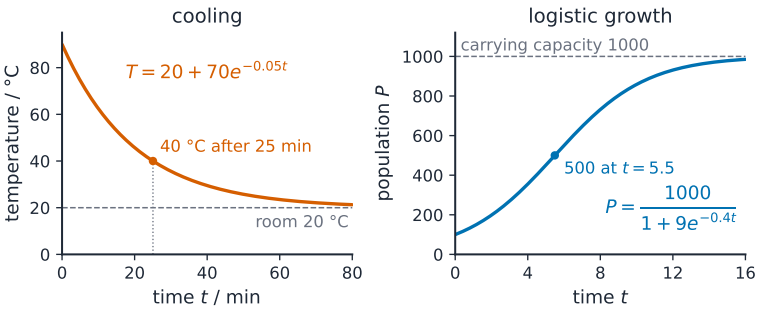
Building a transformation step by step



$y = x^2$
3 left: $y = (x + 3)^2$
stretch $\times 2$: $y = 2(x + 3)^2$
5 down: $y = 2(x + 3)^2 - 5$

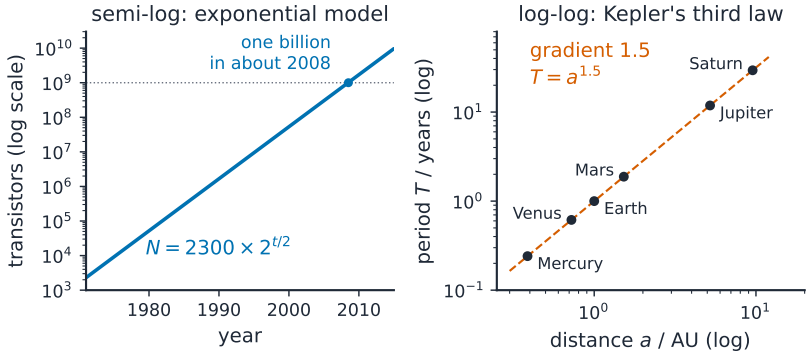
The vertex moves to $(-3, -5)$

Models that level off



- Coffee reaches 40 °C at $t = 20 \ln 3.5 \approx 25.1$.
- Logistic is fastest at half the capacity.

Straight on log axes



- $y = ka^x$ is straight on a **semi-log graph**; $y = ax^n$ is straight on a **log-log graph**, **gradient** n.

Exam tips

> Choose a model from how the quantity changes, then test it.

- Sketch GDC graphs with key points labelled.
- State the **domain** from the context.
- Interpret parameters with units.

Check yourself

Try these

Find the line through $(8, 3)$ perpendicular to $y = 4x - 1$.

A medicine follows $A = 200 \times 0.85^t$. When is it below 50 mg?

Find the largest volume of a box made from a 30 cm square sheet.