

Supported Focus Companion

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpreta- tion · HL

Source-supported focus teaching; remaining qualification requirements are held

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Original teaching and practice · exercises.lol

Contents

1. Number and algebra · supported focus	2
2. Functions · supported focus	15
3. Geometry and trigonometry · supported focus	20
4. Statistics and probability · supported focus	27
5. Calculus · supported focus	38
6. School-supervised mathematical exploration · supported focus	49

Number and algebra · supported focus

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · HL

Scope and prerequisites

Supported HL focus. First assessment 2021; current through 2028. First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.

Prerequisites: read the stated quantities and units, use arithmetic and the model conditions below. Each lesson develops its own method before independent transfer.

These are original or explicitly fictional teaching examples, not actual measurements or completed assessed learner investigations.

Exact arithmetic and estimation

■ Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

■ Choose the mathematical structure

- Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

Shared smallest powers: $2 \times 3^2 = 18$

■ Test a tempting shortcut

- The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

■ Use this in your course

- Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

Percentages, ratio and proportional reasoning

■ Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
 - This lesson studies **multiplier** : A factor that performs a percentage change in one multiplication.
-

■ Choose the mathematical structure

- A p% increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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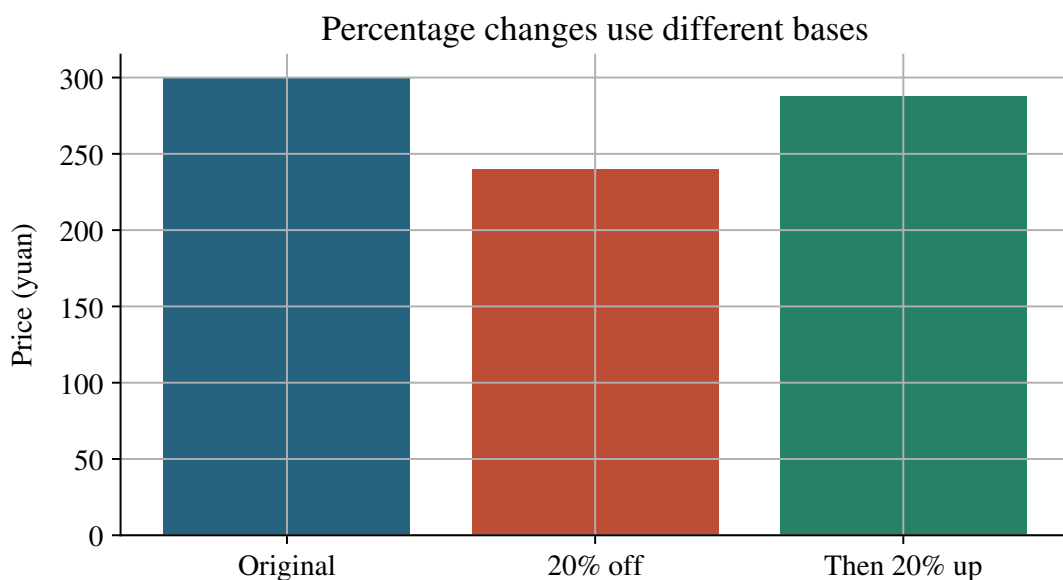
■ Work through a checked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

Let the original price be P. The model is sale price=0.8P. Hence $P=240/0.8=300$. A later 20% increase gives $240 \times 1.2=288$, so the two changes do not cancel.



■ Test a tempting shortcut

- A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A 20% decrease followed by a 20% increase restores the starting price. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For direct proportion use $y=kx$; for inverse proportion use $y=k/x$. Calculate k from a known pair before using a new value. State what you held constant.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

A factor that performs a percentage change in one multiplication. Choose the relationship, show the method, check its assumptions and interpret the result.

Sequences, series and recurrence

■ Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
 - This lesson studies **common ratio** : The constant multiplier between consecutive terms of a geometric sequence.
-

■ Choose the mathematical structure

- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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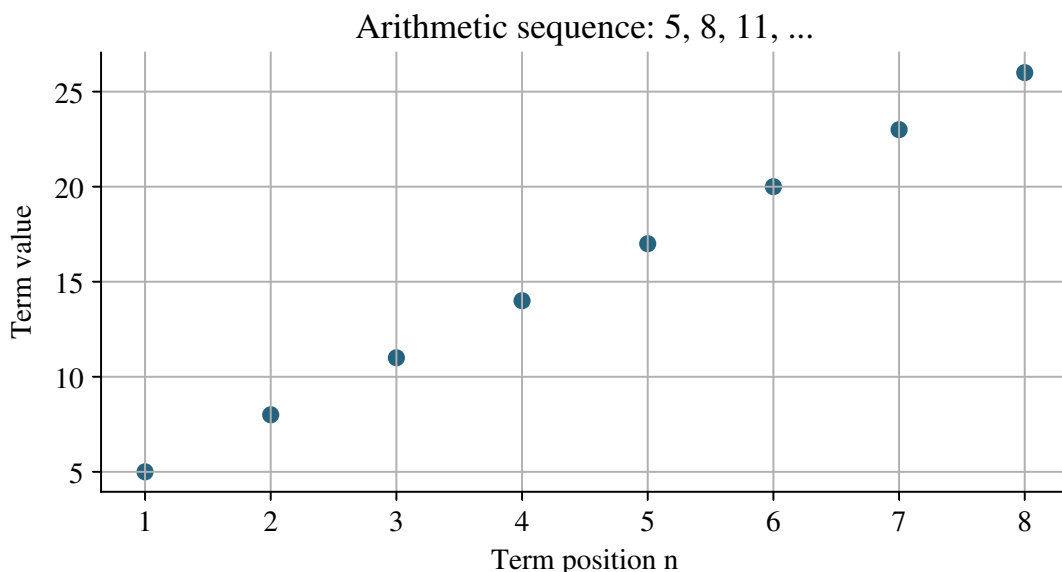
■ Work through a checked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10 + 21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1-1/2) = 24$. For $u_{n+1} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



■ Test a tempting shortcut

- The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

Regression, financial models and residuals

■ Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
 - This lesson studies **residual** : The observed value minus the value predicted by a fitted model.
-

■ Choose the mathematical structure

- A linear model $y=a+bx$ has intercept a and slope b with contextual units. Inspect residuals and the data range. For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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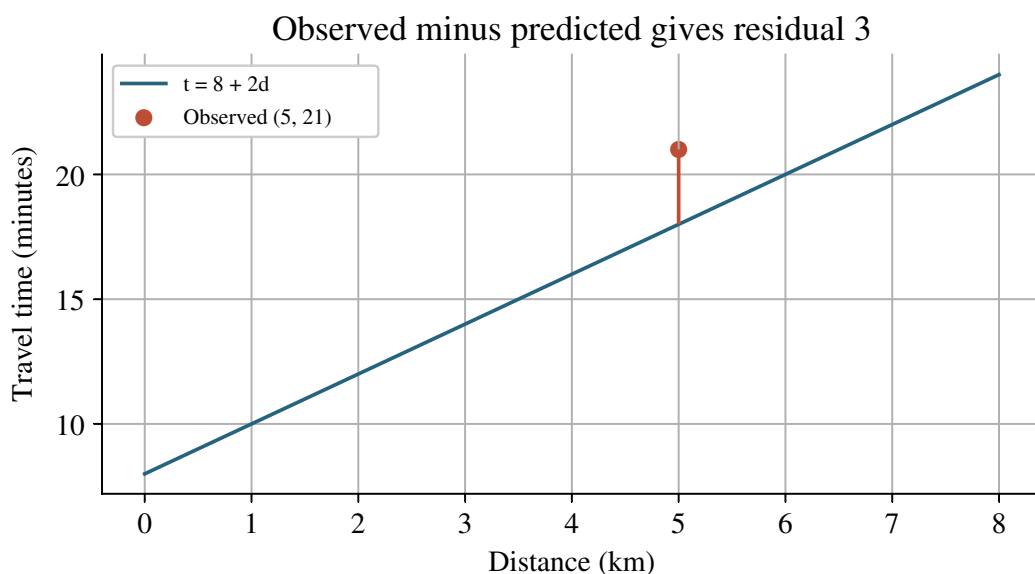
■ Work through a checked case

$$\hat{t} = 8 + 2d, \quad e = t - \hat{t}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

The fitted model $t=8+2d$ predicts 18 minutes at $d=5$ km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes $1000 \times 1.05^3 = 1157.625$ after 3 years.



■ Test a tempting shortcut

- A good fit does not prove a causal mechanism. A correlation coefficient measures linear association, not the gradient. Calculator output must be translated into a model, checked and interpreted.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A high correlation permits reliable extrapolation to any distance. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- In IB AI, record the data source, domain, assumptions and calculator method. Compare an alternative model and judge predictions against residuals. A financial answer must state payment timing and whether interest is compounded.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The observed value minus the value predicted by a fitted model. Choose the relationship, show the method, check its assumptions and interpret the result.

Matrix transformations, inverses and eigenvalues

■ Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
 - This lesson studies **determinant** : A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
-

■ Choose the mathematical structure

- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero. Transform column vectors by left multiplication. The rightmost matrix acts first in a product.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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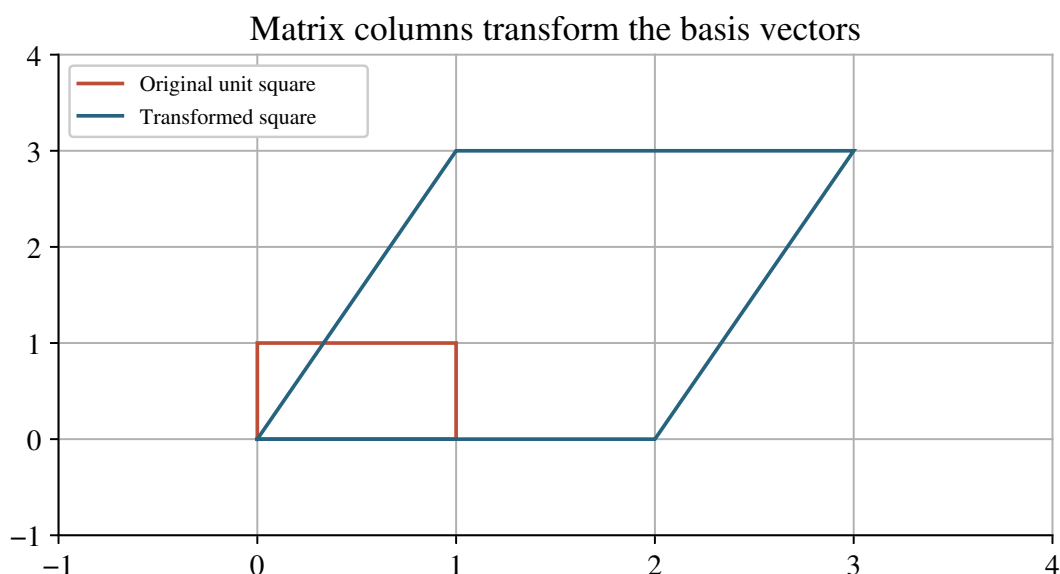
■ Work through a checked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. The characteristic equation is $(2-\lambda)(3-\lambda)=0$, so the eigenvalues are 2 and 3.



■ Test a tempting shortcut

- Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every square matrix has an inverse. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Find an eigenvector by solving $(A - \lambda I)v = 0$ with $v \neq 0$. Explain the geometrical meaning: this vector keeps its line direction under the transformation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor. Choose the relationship, show the method, check its assumptions and interpret the result.

Complex numbers and roots

■ What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
 - This lesson studies **complex conjugate** : The number obtained by reversing the sign of the imaginary part.
-

■ Choose the mathematical structure

- Write $z=a+bi$ with $i^2=-1$. The conjugate is $a-bi$ and z times its conjugate $=a^2+b^2$. The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant. Conjugate roots occur for polynomials with real coefficients.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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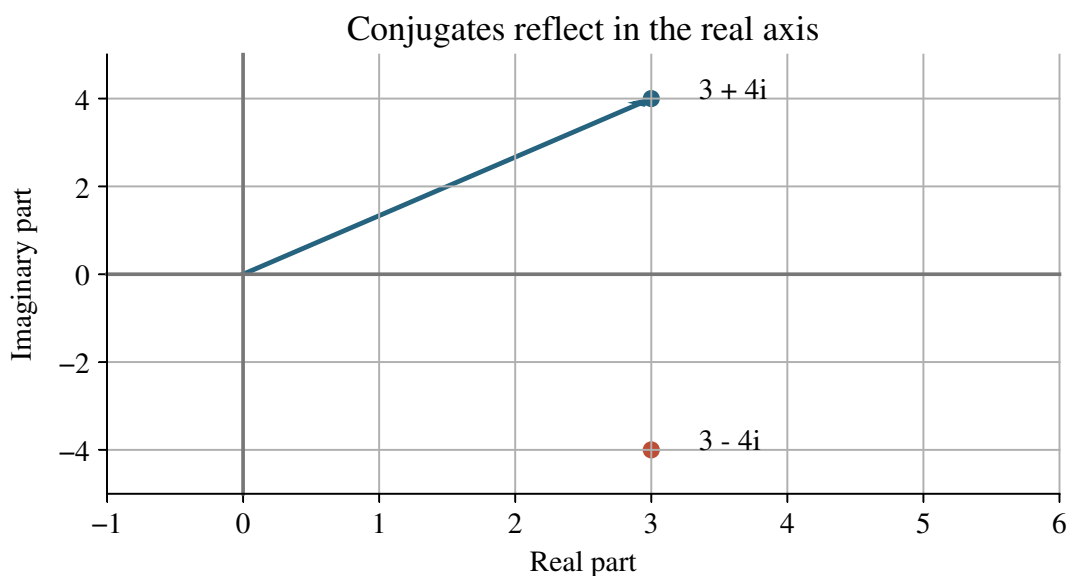
■ Work through a checked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm2i$. Multiplying by the conjugate makes a complex denominator real.



■ Test a tempting shortcut

- The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Draw the real axis horizontally and imaginary axis vertically. For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

Functions · supported focus

International Baccalaureate · IB Diploma · Mathematics:
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Domains, inverses and composition

■ Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
 - This lesson studies **domain** : The set of allowed inputs to a function.
-

■ Choose the mathematical structure

- State the domain and range. For an inverse, first ensure the function is one-to-one on its domain. Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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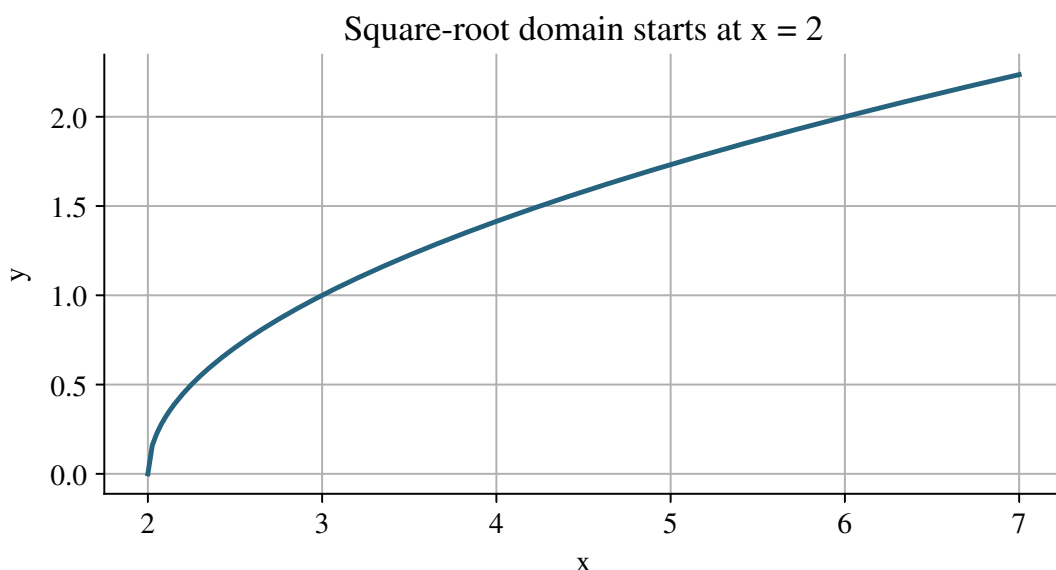
■ Work through a checked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus $f^{-1}(x)=x^2+2$ with $x \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



■ Test a tempting shortcut

- Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every quadratic function on all real numbers has an inverse function. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Check $f(f^{-1}(x))=x$ on the inverse domain. Use a sketch to test whether a horizontal line meets the original graph more than once.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

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Key:

The set of allowed inputs to a function. Choose the relationship, show the method, check its assumptions and interpret the result.

Exponentials, logarithms and modelling

■ Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
 - This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.
-

■ Choose the mathematical structure

- For $y = Ae^{kt}$, k is a proportional rate. Taking logs gives $\ln y = \ln A + kt$. Logarithms require positive arguments, and $\log(x+y)$ is not $\log x + \log y$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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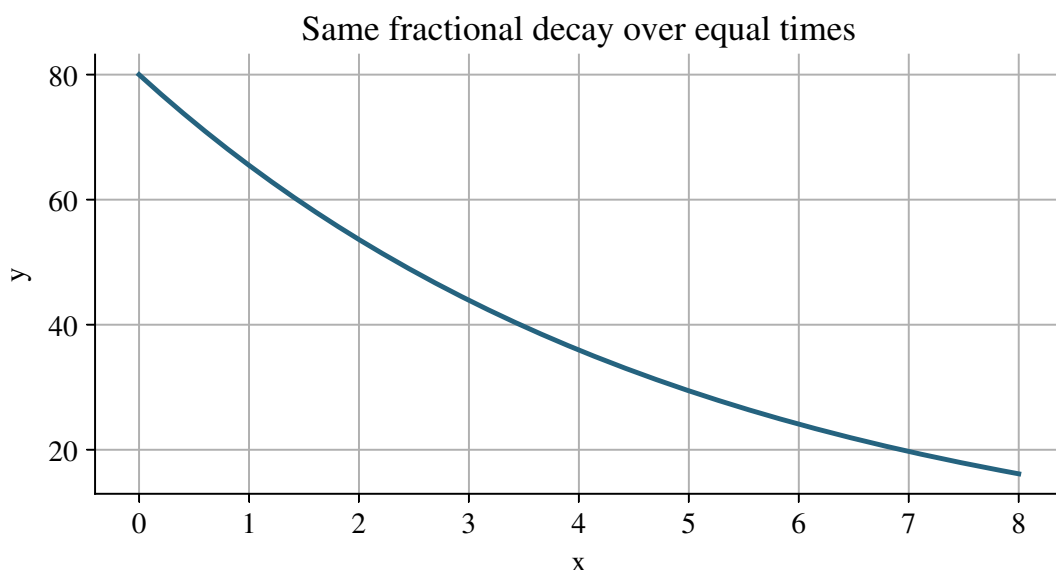
■ Work through a checked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

If $y = 80e^{-0.2t}$, $y = 40$ gives $e^{-0.2t} = 1/2$. Hence $t = \ln 2 / 0.2 \approx 3.466$. For $3^x = 20$, $x = \ln 20 / \ln 3$. Plotting $\ln y$ against t linearises this exponential model.



■ Test a tempting shortcut

- A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative k describes decay; a negative starting amount usually contradicts the context.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For positive x and y , $\ln(x+y) = \ln x + \ln y$. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Compare actual observations with the model. Systematic departures may indicate changing conditions. In a report, explain what the rate and initial value mean, rather than giving a bare equation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The time for a decaying quantity to fall to half its initial value. Choose the relationship, show the method, check its assumptions and interpret the result.

Geometry and trigonometry · supported focus

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Angle reasoning, similarity and mensuration

■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
 - This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.
-

■ Choose the mathematical structure

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 . State angle reasons explicitly. A circle's tangent is perpendicular to the radius at the contact point.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

■ Work through a checked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

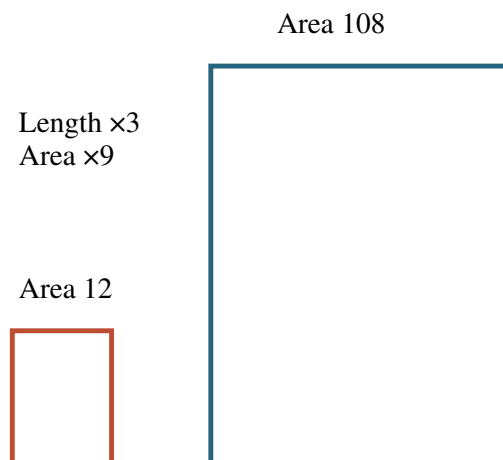
- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108 \text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216 \text{ cm}^3$. A cylinder with $r=3, h=5$ has volume

$$\pi r^2 h = 45\pi.$$

A length scale factor is squared for area



■ Test a tempting shortcut

- Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Doubling every length of a solid doubles its volume. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion. A scale drawing is evidence only when the task permits measurement.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The multiplier that relates corresponding lengths in similar shapes. Choose the relationship, show the method, check its assumptions and interpret the result.

Right triangles and non-right triangles

■ Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
 - This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.
-

■ Choose the mathematical structure

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite}/\text{hypotenuse}$, $\cos\theta = \text{adjacent}/\text{hypotenuse}$ and $\tan\theta = \text{opposite}/\text{adjacent}$. For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

■ Work through a checked case

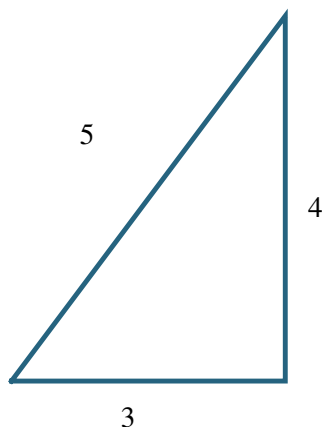
$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan\theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



■ Test a tempting shortcut

- Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Pythagoras applies to every triangle, including triangles without a right angle. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule. Explain why the chosen triangle contains the required length or angle.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

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Key:

The side opposite the right angle in a right-angled triangle. Choose the relationship, show the method, check its assumptions and interpret the result.

Vectors and transformation geometry

■ Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
 - This lesson studies **resultant** : The vector sum representing the combined displacement or force.
-

■ Choose the mathematical structure

- Add corresponding vector components. A displacement from A to B is $\vec{OB}-\vec{OA}$. Parallel vectors are scalar multiples. A vector has magnitude and direction; a distance is a scalar.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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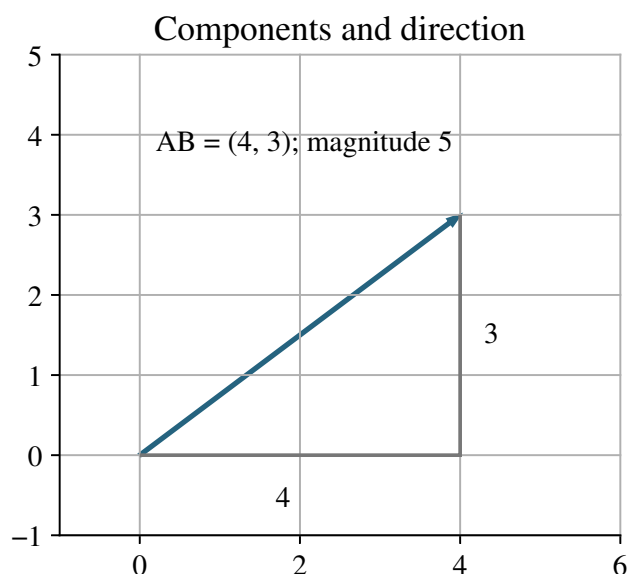
■ Work through a checked case

$$\vec{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

With $\mathbf{a}=(4,1)$ and $\mathbf{b}=(1,3)$, $\mathbf{a}+\mathbf{b}=(5,4)$. Their scalar product is $\mathbf{a} \cdot \mathbf{b}=4 \times 1+1 \times 3=7$. The magnitude of $\mathbf{a}$ is $\sqrt{17}$. For $\mathbf{A}=(1,2)$, $\mathbf{B}=(5,5)$, $\mathbf{AB}=(4,3)$ and $|\mathbf{AB}|=5$.



■ **Test a tempting shortcut**

- The order of subtraction matters: $BA = -AB$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

AB and BA always have the same components. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- For a translation, every point uses the same displacement. For advanced line problems, distinguish the position vector from a direction vector and solve parameter equations consistently.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

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Key:

The vector sum representing the combined displacement or force. Choose the relationship, show the method, check its assumptions and interpret the result.

Statistics and probability · supported focus

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · HL

Scope and prerequisites

Supported HL focus. First assessment 2021; current through 2028. First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.

Prerequisites: read the stated quantities and units, use arithmetic and the model conditions below. Each lesson develops its own method before independent transfer.

These are original or explicitly fictional teaching examples, not actual measurements or completed assessed learner investigations.

Data summaries, histograms and interpretation

■ Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
 - This lesson studies **frequency density** : Frequency divided by class width, used as histogram height.
-

■ Choose the mathematical structure

- Compare an appropriate average and spread in context. A histogram uses area for frequency, so height=frequency/class width. Grouped estimates assume representative values within intervals.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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■ Work through a checked case

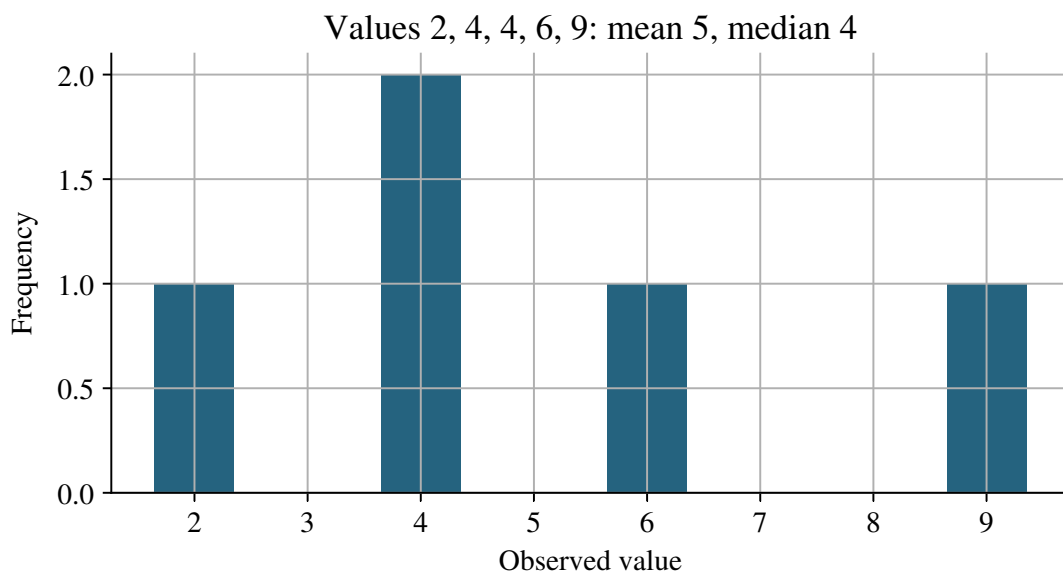
$$\bar{x} = \frac{\sum x_i}{n}, \quad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a

larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.



■ Test a tempting shortcut

- The tallest histogram bar need not contain the most observations. A grouped mean is an estimate. Correlation does not prove causation, and extrapolation extends beyond the observed range.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A histogram bar's height always equals its frequency, even with unequal class widths. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Choose a display that fits the data type. Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

■ Use this in your course

- Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

Frequency divided by class width, used as histogram height. Choose the relationship, show the method, check its assumptions and interpret the result.

Probability, trees and conditional reasoning

■ What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
 - This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.
-

■ Choose the mathematical structure

- Multiply along a tree branch and add disjoint branches. With replacement, the composition stays fixed. Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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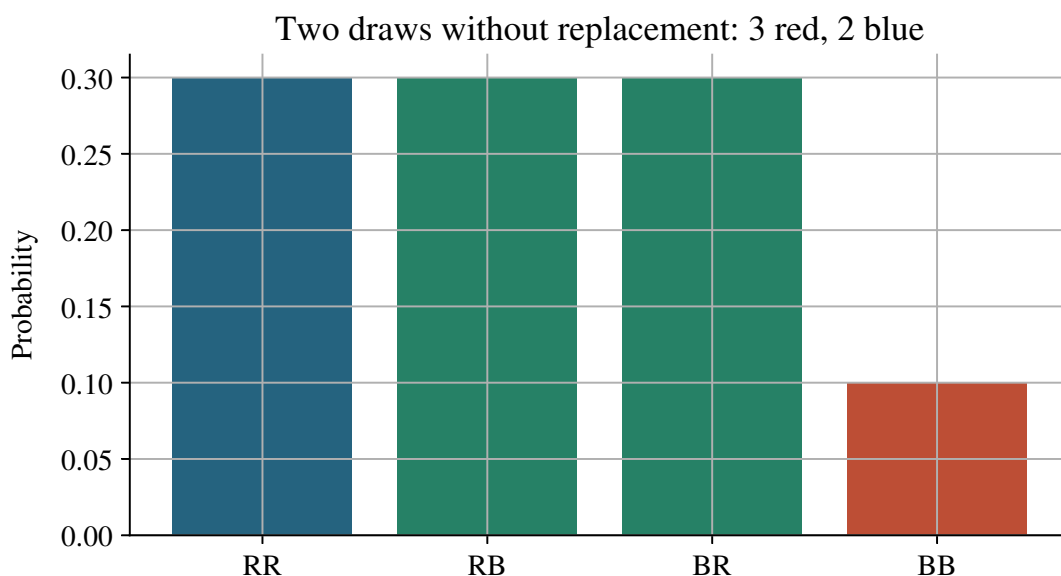
■ Work through a checked case

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



■ Test a tempting shortcut

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Mutually exclusive events with positive probabilities must be independent. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- A two-way table makes the restricted denominator visible. Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

The probability of an event after restricting the sample space to a stated condition. Choose the relationship, show the method, check its assumptions and interpret the result.

Binomial, normal and Poisson models

■ What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
 - This lesson studies **expected value** : The probability-weighted mean of a random variable.
-

■ Choose the mathematical structure

- A binomial model needs fixed n , independent trials, two outcomes and constant p . $E(X)=np$ and $\text{Var}(X)=np(1-p)$. For a normal model use $z=(x-\mu)/\sigma$. A Poisson model describes counts with a constant rate and appropriate independence assumptions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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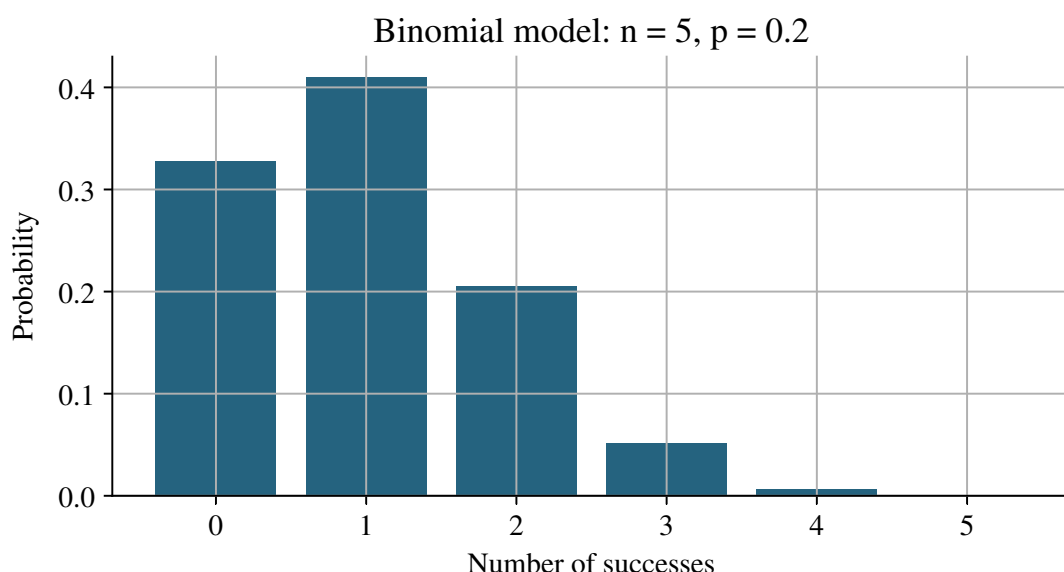
■ Work through a checked case

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad X \sim B(n, p)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For X binomial(5,0.2), $P(X=0)=0.8^5=0.32768$, $E(X)=1$ and $\text{Var}(X)=0.8$. For a normal quantity with $\mu=100, \sigma=15$, the value 130 has $z=2$. A Poisson mean of 3 per hour gives mean 6 over two hours.



■ Test a tempting shortcut

- Not every count is binomial: changing p or dependence can invalidate it. For a continuous variable, the probability of one exact value is zero. Continuity correction matters when approximating a discrete distribution by a normal one.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every count of successes has a binomial distribution regardless of dependence. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Write the event as an inequality before using calculator distribution functions. Distinguish $P(X < k)$, $P(X \leq k)$ and a tail complement. State assumptions in context.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

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Key:

The probability-weighted mean of a random variable. Choose the relationship, show the method, check its assumptions and interpret the result.

Hypothesis testing and contextual conclusions

■ Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
 - This lesson studies **significance level** : The chosen probability threshold for rejecting a null hypothesis.
-

■ Choose the mathematical structure

- State H_0 and H_1 in population parameters before inspecting the outcome. Calculate the appropriate tail probability under H_0 . Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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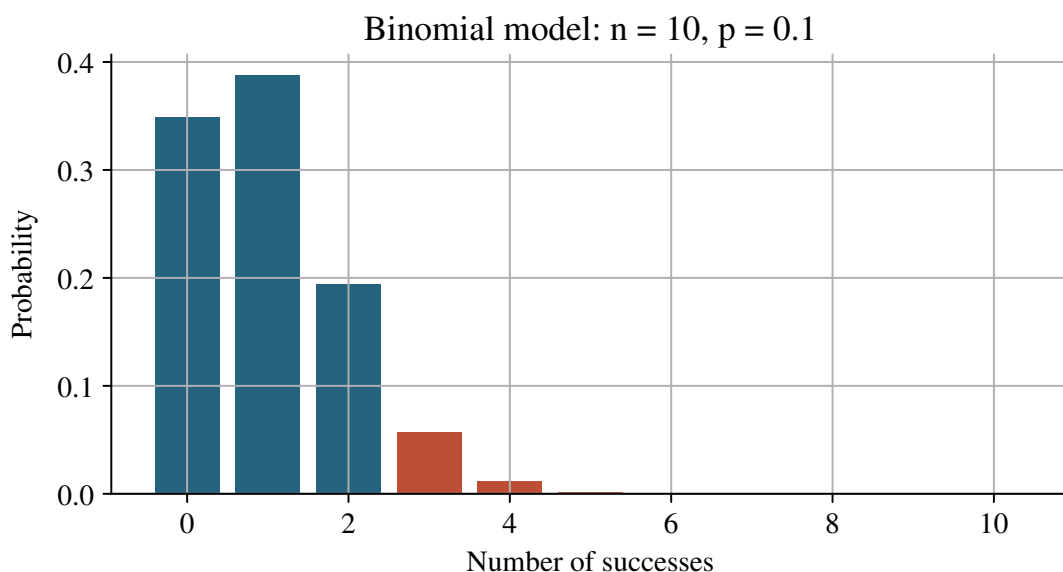
■ Work through a checked case

$$H_0 : p = p_0, \quad H_1 : p > p_0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $H_0: p=0.1$ and $H_1: p>0.1$ with $n=10$, observing at least 3 defects has probability $1-(0.9^{10}+10 \times 0.1 \times 0.9^9+45 \times 0.1^2 \times 0.9^8) \approx 0.070191$. At 5%, this is insufficient evidence that the defect rate exceeds 10%.



■ Test a tempting shortcut

- Failing to reject H_0 is not proof that H_0 is true. Choose the tail from H_1 , not from whichever tail gives a small result. Statistical significance does not measure the practical size of an effect.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Failing to reject a null hypothesis proves it is true. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Finish with a sentence about the population and the original claim. State the model's assumptions and consider whether the sampling procedure supports them.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

■ Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The chosen probability threshold for rejecting a null hypothesis. Choose the relationship, show the method, check its assumptions and interpret the result.

Confidence intervals, chi-square and regression

■ How uncertain is the sample mean?

- Two samples give different means. We need a statement about sampling uncertainty, not a promise that one sample equals the population.
 - This lesson studies **confidence interval** : An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate.
-

■ Choose the mathematical structure

- For a normal mean with known σ , a 95% interval is $\text{sample mean} \pm 1.96\sigma/\sqrt{n}$. For chi-square, $\sum (\text{observed}-\text{expected})^2/\text{expected}$ with the correct degrees of freedom. Choose a test that fits the data and assumptions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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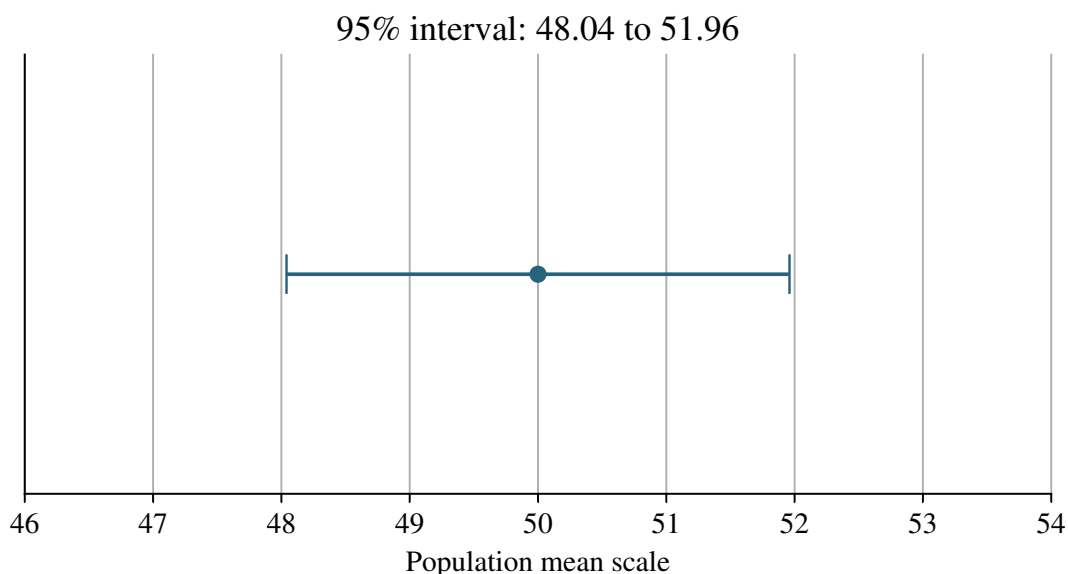
■ Work through a checked case

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}, \quad \chi^2 = \sum \frac{(O - E)^2}{E}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

With $\text{mean}=50, \sigma=10, n=100$, standard error=1 and the 95% interval is [48.04, 51.96]. For observed counts 30 and 20 against expected 25 and 25, $\text{chi-square}=25/25+25/25=2$, before deciding degrees of freedom and the rejection threshold.



■ Test a tempting shortcut

- A 95% confidence procedure does not give a 95% probability that a fixed population mean moves within this one interval. Expected counts and estimated parameters affect the validity and degrees of freedom of chi-square tests.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A strong correlation alone establishes a causal relationship. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Report uncertainty in the context. In regression, inspect residuals, avoid unjustified extrapolation, and distinguish a fitted relationship from a causal explanation. S3 and IB AI HL require different verified scope boundaries.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

■ Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate. Choose the relationship, show the method, check its assumptions and interpret the result.

Calculus · supported focus

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · HL

Scope and prerequisites

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Prerequisites: read the stated quantities and units, use arithmetic and the model conditions below. Each lesson develops its own method before independent transfer.

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Derivatives and stationary points

■ What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
 - This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.
-

■ Choose the mathematical structure

- For $y = ax^n$, $dy/dx = anx^{n-1}$. A stationary point satisfies $dy/dx = 0$. Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

■ Work through a checked case

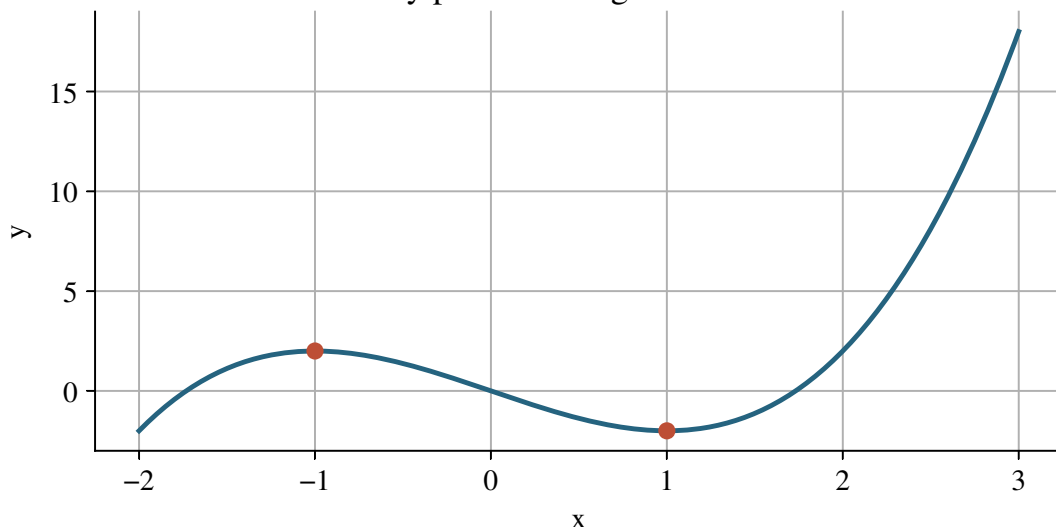
$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $y = x^3 - 3x$, $dy/dx = 3x^2 - 3$. At $x = 1$, the gradient is 0 and $y = -2$. The second derivative is $6x$, positive at $x = 1$, so this is a local minimum. At $x = -1$, $y = 2$ and the second derivative is negative, giving a local maximum.

Stationary points and signed accumulation



■ Test a tempting shortcut

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there. At advanced level, connect the derivative to rates and optimization with a valid domain.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

Root finding and numerical integration

■ How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
 - This lesson studies **iteration** : A repeated update in which each new approximation is calculated from the previous one.
-

■ Choose the mathematical structure

- A continuous function with opposite signs at two endpoints has a root between them. Newton's method uses $x_{\text{next}} = x - f(x)/f'(x)$, with a nonzero derivative. The trapezium rule approximates a definite integral using endpoint heights.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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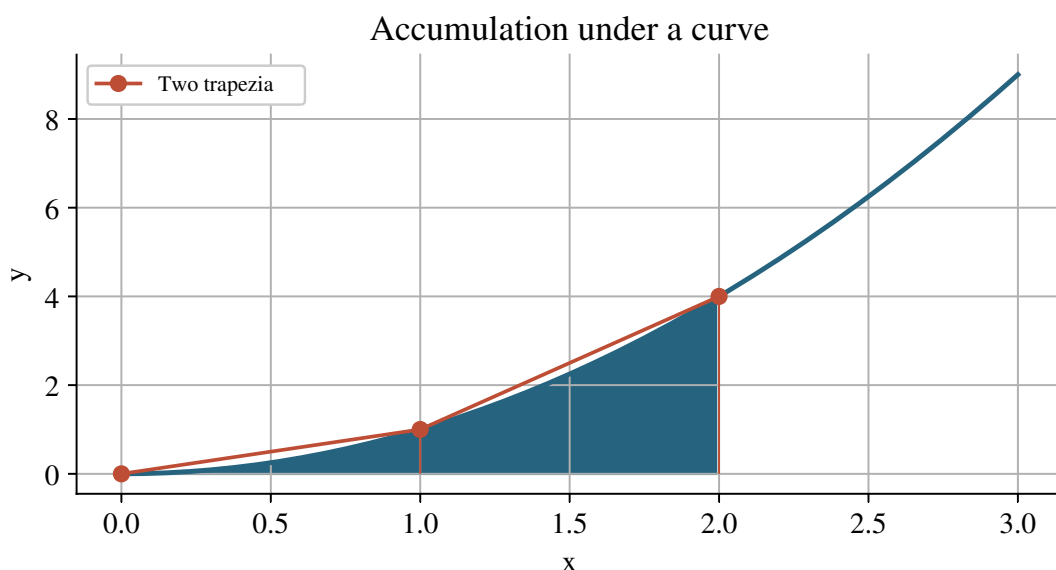
■ Work through a checked case

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $f(x) = x^2 - 2$ and $x_0 = 1.5$, Newton gives $x_1 = 1.5 - (2.25 - 2)/3 = 1.4166667$. With $y = x^2$ on $[0, 2]$ and two equal strips, $h = 1$ and trapezium area $= (1/2)(0 + 2 \times 1 + 4) = 3$; the exact area is $8/3$.



■ Test a tempting shortcut

- A sign change across a discontinuity does not prove a root. Iteration can diverge or cycle. The trapezium rule's overestimate or underestimate depends on curvature, not just whether the function increases.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every sign change proves a root, including one across a discontinuity. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Give a stopping criterion and report sensible accuracy. Confirm the approximated root by a sign bracket around the stated rounded answer; explain any failure of the chosen iteration.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

Chain, product, quotient and implicit differentiation

■ Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** : The rule that multiplies the outer derivative by the inner derivative for a composite function.
-

■ Choose the mathematical structure

- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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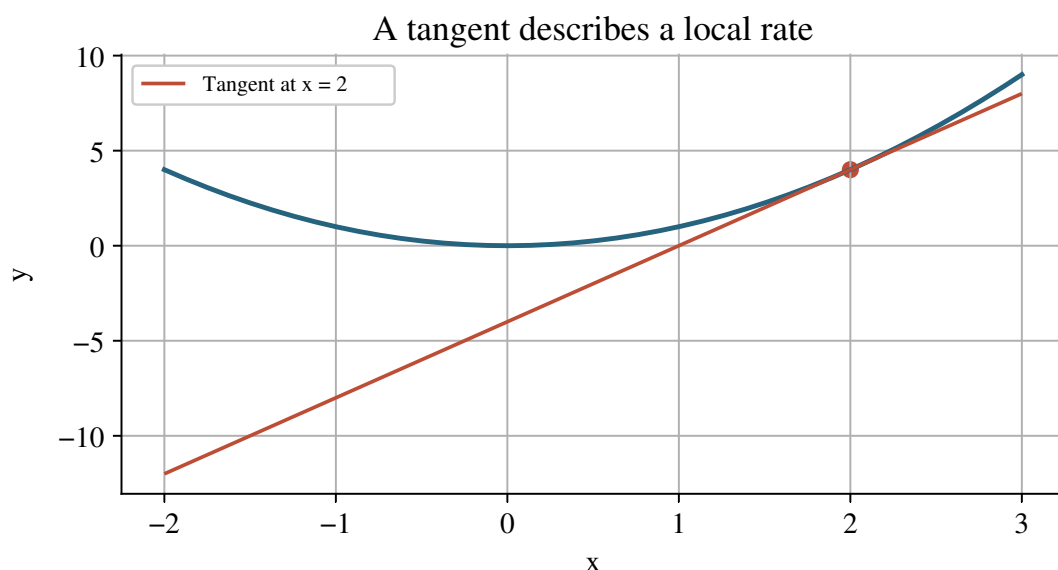
■ Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



■ Test a tempting shortcut

- A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The derivative of $u(x)v(x)$ is always u prime times v prime. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

Substitution, parts and partial fractions

■ Can we integrate the two factors separately?

- A simple-looking product such as $x e^x$ cannot be integrated by separately integrating each factor.
 - This lesson studies **integration by parts** : An integration method based on the derivative of a product.
-

■ Choose the mathematical structure

- Use substitution when an inner derivative appears as a factor. By parts, $\int u v' = uv - \int u' v$. For a rational function, divide first if needed, then decompose into partial fractions.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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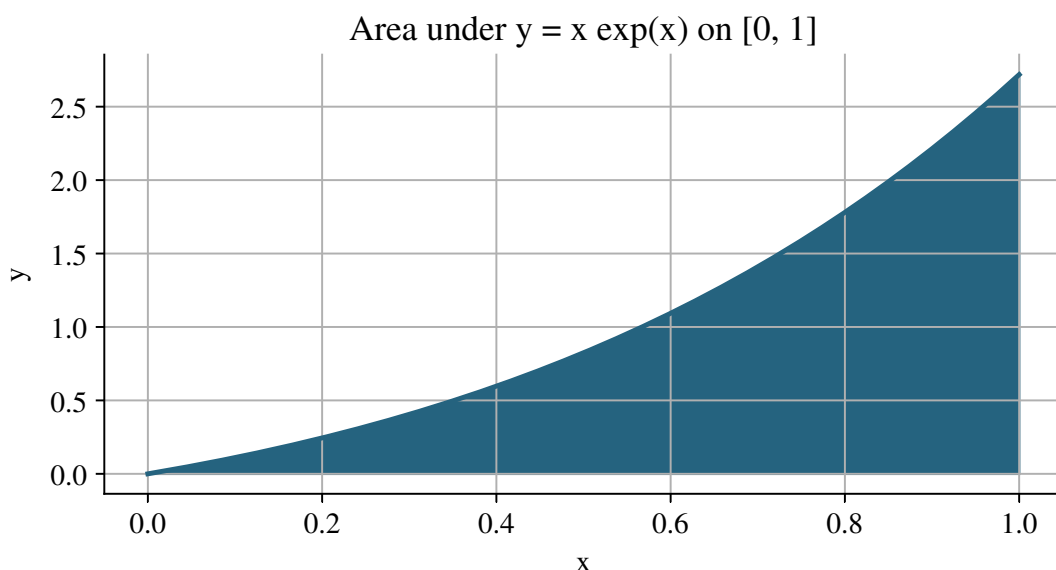
■ Work through a checked case

$$\int u v' dx = uv - \int u' v dx$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For integral $\int x e^x dx$, take $u=x$ and $v' = e^x$. Then the integral is $x e^x - e^x + C$. For integral $\int 2x/(x^2+1) dx$, substitute $w=x^2+1$, $dw=2x dx$; the answer is $\ln(x^2+1)+C$.



■ Test a tempting shortcut

- Choosing u and v prime well matters: the remaining integral should become simpler. In a definite substitution, either change the limits or return to x before applying the original limits.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The integral of a product equals the product of its separate integrals. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Check by differentiating. For volumes of revolution around the x -axis use $V = \pi \int y^2 dx$; do not confuse the square of a function with the integral of the function.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

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Key:

An integration method based on the derivative of a product. Choose the relationship, show the method, check its assumptions and interpret the result.

Differential equations and numerical solutions

■ What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
 - This lesson studies **initial condition** : A specified value that selects a particular solution from a family.
-

■ Choose the mathematical structure

- For $dy/dx=ky$, separate variables: $dy/y=k dx$, giving $y=Ae^{(kx)}$. Use an initial condition to find A. Euler's method takes $y_{next}=y+h f(x,y)$, with a chosen step h.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
-

■ Work through a checked case

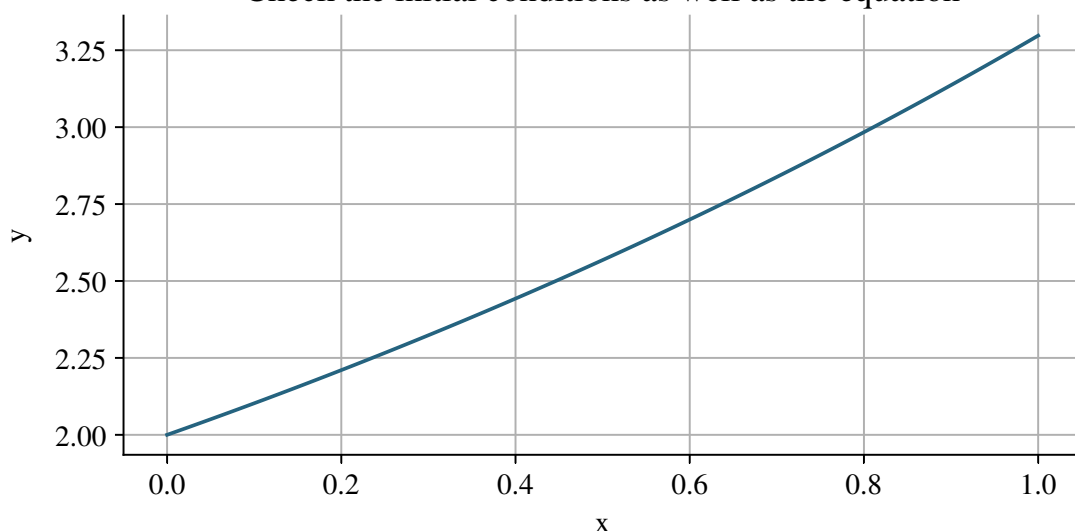
$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

With $dy/dx=0.5y$ and $y(0)=2$, $y=2e^{(0.5x)}$. Euler with $h=0.2$ gives $y(0.2)\approx 2+0.2\times 1=2.2$ and $y(0.4)\approx 2.2+0.2\times 1.1=2.42$. The exact second value is about 2.4428.

Check the initial conditions as well as the equation



■ Test a tempting shortcut

- An initial condition determines the integration constant; it is not a replacement for integrating. Dividing by y can omit an equilibrium solution $y=0$. Euler accuracy depends on step size and the equation.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every step size gives exactly the same Euler approximation. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions. This is Further Pure scope; check the named unit.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
-

■ **Use this in your course**

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Key:

A specified value that selects a particular solution from a family. Choose the relationship, show the method, check its assumptions and interpret the result.

School-supervised mathematical exploration · supported focus

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · HL

Scope and prerequisites

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Planning a mathematical exploration

■ What makes an investigation mathematical?

- A student wants to predict queue waiting time at the school canteen. A useful exploration needs a question, data and mathematics the student can explain.
- This lesson studies **assumption** : A condition adopted to make a mathematical model possible and whose effect should be evaluated.

■ Choose the mathematical structure

- Frame a focused question with measurable variables and a feasible method. Keep a dated record of decisions, sources and mathematical reasoning. Use technology to support analysis, and explain every important output in your own words.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

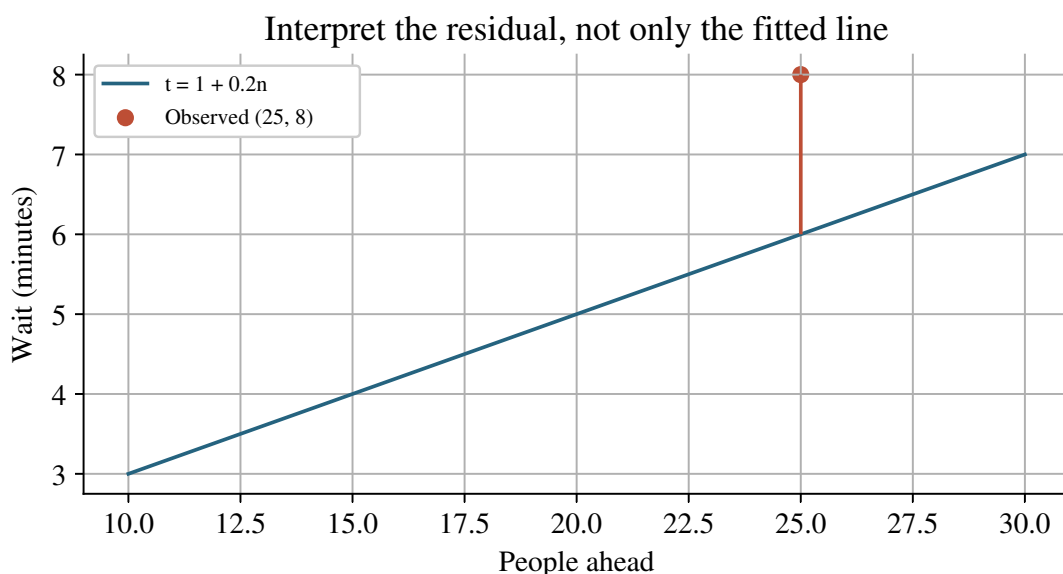
■ Work through a checked case

$$\hat{t} = 1 + 0.2n, \quad e = t - \hat{t}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

A pilot records mean waiting times of 3,5,7 minutes when 10,20,30 people are ahead. A first model is $t=1+0.2n$. At $n=25$ it predicts 6 minutes. An actual wait of 8 gives residual 2; investigate varying service rates rather than hiding the mismatch.



■ Test a tempting shortcut

- Do not copy a sample exploration, invent data or manufacture personal engagement. A prediction is not evidence until it is compared with observations. Consent and privacy matter when collecting school data.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The school-supervised mathematical exploration can be replaced by a timed written mock exam. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- The school supervises topic approval, checkpoints, authenticity and assessment. For the current 2021 course use the current school's guide and criteria; the 2029 inquiry criteria belong to a different assessment version.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

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Key:

A condition adopted to make a mathematical model possible and whose effect should be evaluated. Choose the relationship, show the method, check its assumptions and interpret the result.