

Exercise walk-through

5.2

Root finding and numerical integration

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation

5.2 Root finding and numerical integration

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS iteration

Objective

Root finding and numerical integration. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** : A repeated update in which each new approximation is calculated from the previous one.

■ Choose the mathematical structure

- A continuous function with opposite signs at two endpoints has a root between them. Newton's method uses $x_{\text{next}} = x - f(x)/f'(x)$, with a nonzero derivative. The trapezium rule approximates a definite integral using endpoint heights.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

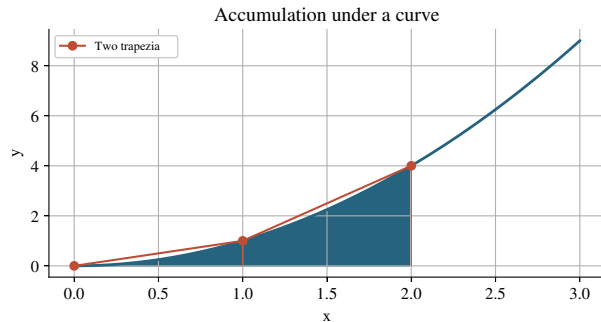
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $f(x)=x^2-2$ and $x_0=1.5$, Newton gives $x_1=1.5-(2.25-2)/3=1.4166667$. With $y=x^2$ on $[0,2]$ and two equal strips, $h=1$ and trapezium area $=(1/2)(0+2\times 1+4)=3$; the exact area

is $8/3$.



■ **Test a tempting shortcut**

- A sign change across a discontinuity does not prove a root. Iteration can diverge or cycle. The trapezium rule's overestimate or underestimate depends on curvature, not just whether the function increases.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every sign change proves a root, including one across a discontinuity. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- Give a stopping criterion and report sensible accuracy. Confirm the approximated root by a sign bracket around the stated rounded answer; explain any failure of the chosen iteration.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

- Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1·2.3

2.1 After one Newton step for x^2-2 from $x_0=1.5$, find x_1 . Show the relation, substitution and units. [2]

2.2 Use two strips for the trapezium estimate of integral x^2 from 0 to 2. Show the relation,

2.3 Find the exact integral of x^2 from 0 to 2. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 After one Newton step for x^2-2 from $x_0=1.5$, find x_1 . Show the relation, substitution and units. [2]

2.1 $x_1=1.5-(1.5^2-2)/(2\times 1.5)=17/12\approx 1.416667$.

2.2 Use two strips for the trapezium estimate of integral x^2 from 0 to 2. Show the relation,

2.2 $h=1$ with heights 0,1,4 gives $(1/2)(0+2\times 1+4)=3$.

2.3 Find the exact integral of x^2 from 0 to 2. Show the relation, substitution and units. [2]

2.3 The exact integral is $[x^3/3]$ from 0 to 2 $=8/3$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every sign change proves a root, including one across a discontinuity. [2]

2.5 Which description correctly defines iteration? Choose one answer.

- **A** A repeated update in which each new approximation is calculated from the previous one.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every sign change proves a root, including one across a discontinuity. [2]

2.4 False. A sign change across a discontinuity does not prove a root. Iteration can diverge or cycle. The trapezium rule's overestimate or underestimate depends on curvature, not just whether the function increases.

2.5 Which description correctly defines iteration? Choose one answer.

- **A** A repeated update in which each new approximation is calculated from the previous one.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

2.5 A A repeated update in which each new approximation is calculated from the previous one.

Exercise 3.1

3.1 Apply one Newton step to $f(x)=x^3-2x+2$ from $x_0=0$ and then one more. What happens, and what does it teach about convergence? [4]

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3.1 $x_1=0-2/(-2)=1$; $x_2=1-1/1=0$. It cycles 0, 1 rather than converging. Newton requires suitable starting values/derivatives; use a bracketed method or inspect the function rather than assuming every iteration converges.