

Exercise walk-through

1.5

Matrix transformations, inverses and eigenvalues

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation

1.5 Matrix transformations, inverses and eigenvalues

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS determinant

Objective

Matrix transformations, inverses and eigenvalues. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** : A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

■ **Choose the mathematical structure**

- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero. Transform column vectors by left multiplication. The rightmost matrix acts first in a product.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

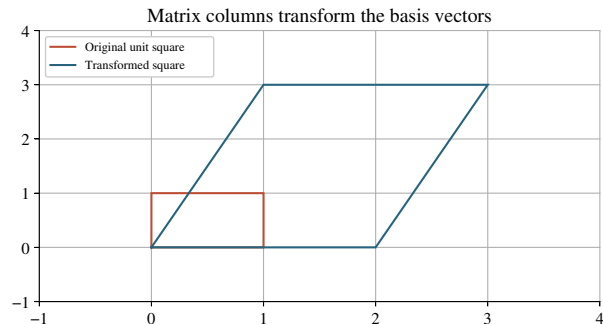
■ **Work through a checked case**

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. The characteristic equation is $(2-\lambda)(3-\lambda)=0$, so the eigenvalues are 2 and 3.



■ Test a tempting shortcut

- Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every square matrix has an inverse. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Find an eigenvector by solving $(A - \lambda I)v = 0$ with $v \neq 0$. Explain the geometrical meaning: this vector keeps its line direction under the transformation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor. Choose the relationship, show the method, check its assumptions

and interpret the result.

Exercises 2.1·2.3

2.1 Find the determinant of $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$. Show the relation, substitution and units. [2]

2.2 Find the x-component after that matrix transforms $(1,2)$. Show the relation, substitution and units. [2]

2.3 Find the larger eigenvalue of that matrix. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

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2.1 The determinant is $2 \times 3 - 1 \times 0 = 6$.

2.2 Find the x-component after that matrix transforms $(1,2)$. Show the relation, substitution and units. [2]

2.2 The first transformed coordinate is $2 \times 1 + 1 \times 2 = 4$.

2.3 Find the larger eigenvalue of that matrix. Show the relation, substitution and units. [2]

2.3 This triangular matrix has eigenvalues given by its diagonal entries 2 and 3, so the larger is 3.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every square matrix has an inverse.[2]

2.5 Which description correctly defines determinant? Choose one answer.

- **A** A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every square matrix has an inverse.[2]

2.4 False. Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

2.5 Which description correctly defines determinant? Choose one answer.

- **A** A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

2.5 A A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

Exercise 3.1

3.1 For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, find an eigenvector for each eigenvalue and check whether $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector. [4]

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3.1 For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, find an eigenvector for each eigenvalue and check whether $(1, 1)$ is an eigenvector. [4]

3.1 λ_2 : eigenvector $(1, 0)$; λ_3 : $(1, 1)$. $A(1, 1) = (3, 3) = 3(1, 1)$. Nonzero scalar multiples also work; the zero vector is excluded.