

Exercise walk-through

1.1

Exact arithmetic and estimation

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation

1.1 Exact arithmetic and estimation

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS prime factor

Objective

Exact arithmetic and estimation. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

■ Choose the mathematical structure

- Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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■ **Work through a checked case**

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{Shared smallest powers: } 2 \times 3^2 = 18$$

■ **Test a tempting shortcut**

- The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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■ **Use this in your course**

- Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1-2.3

2.1 Find the HCF of 48 and 60. Show the relation, substitution and units. [2]

2.2 Find the LCM of 8 and 12. Show the relation, substitution and units. [2]

2.3 Work out $\frac{3}{4} + \frac{5}{6}$ as a decimal. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find the HCF of 48 and 60. Show the relation, substitution and units. [2]

2.1 $48=2^4 \times 3$ and $60=2^2 \times 3 \times 5$. The shared smallest powers give $\text{HCF}=2^2 \times 3=12$.

2.2 Find the LCM of 8 and 12. Show the relation, substitution and units. [2]

2.2 $8=2^3$ and $12=2^2 \times 3$. The largest prime powers give $\text{LCM}=2^3 \times 3=24$.

2.3 Work out $\frac{3}{4} + \frac{5}{6}$ as a decimal. Show the relation, substitution and units. [2]

2.3 Use denominator 12: $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}$.

Exercises 2.4-2.5

2.4 Decide whether this claim is true and justify: The HCF of two positive integers is always larger than either integer. [2]

2.5 Which description correctly defines prime factor? Choose one answer.

- **A** A prime number that divides the integer exactly.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: The HCF of two positive integers is always larger than either integer. [2]

2.4 False. The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.

2.5 Which description correctly defines prime factor? Choose one answer.

- **A** A prime number that divides the integer exactly.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

2.5 A A prime number that divides the integer exactly.

Exercise 3.1

3.1 A supplier has 84 pencils and 126 pens. Find the greatest number of identical bags without leftovers and the contents of each; then explain why the LCM answers a different question. [4]

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3.1 $HCF=42$, each bag 2 pencils and 3 pens. $LCM=252$ is a common multiple, useful for matching repeated quantities, not the greatest number of bags dividing both stocks.