

Statistics and probability

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · HL

Scope and prerequisites

This handout covers selected HL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Data summaries, histograms and interpretation

■ Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** : Frequency divided by class width, used as histogram height.

■ The method

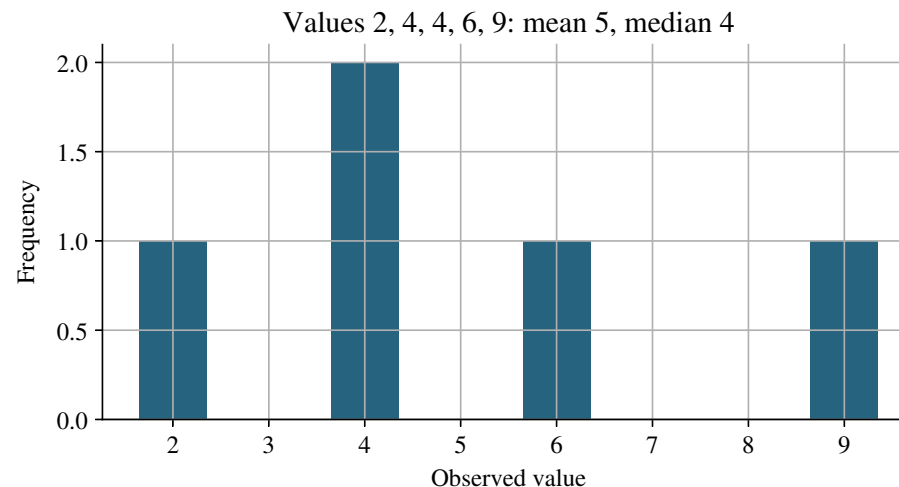
- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

■ A worked case

$$\bar{x} = \frac{\sum x_i}{n}, \quad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

Example:

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.



For 2, 4, 4, 6, 9: mean = $25 \div 5 = 5$, median = 4

■ Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

Not true: A histogram bar's height always equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

■ Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

■ In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Frequency density:** Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

■ **Write from a mean of 9**

- A class of width 2 has frequency 22.
- The values 5, 8, 8, 11 and 13 have a mean and a median. Bar height is not always the frequency.

Probability, trees and conditional reasoning

■ **What changes after the first draw?**

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

■ **The method**

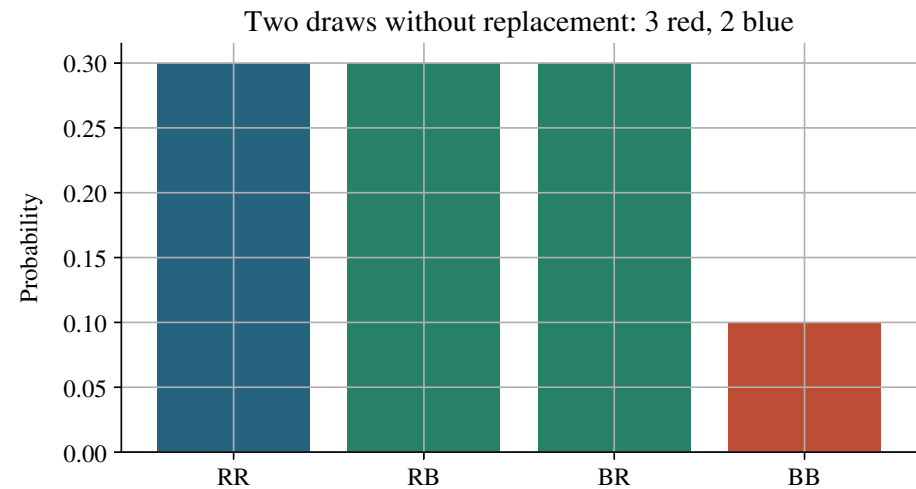
- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.

■ **A worked case**

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Example:

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



Without replacement: $P(RR) = 3/5 \times 2/4 = 0.3$ and $P(BB) = 2/5 \times 1/4 = 0.1$

■ **Where it goes wrong**

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

Not true: Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B) = 0 \neq P(A)P(B)$.

■ **Applying it**

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

■ **In your course**

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Conditional probability:** The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

■ Write from a conditional probability of 0.72

- $P(A \text{ and } B)$ is 0.36 and $P(B)$ is 0.50.
- Find $P(A \text{ given } B)$. Mutually exclusive events that can happen are not independent.

Binomial, normal and Poisson models

■ What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** : The probability-weighted mean of a random variable.

■ The method

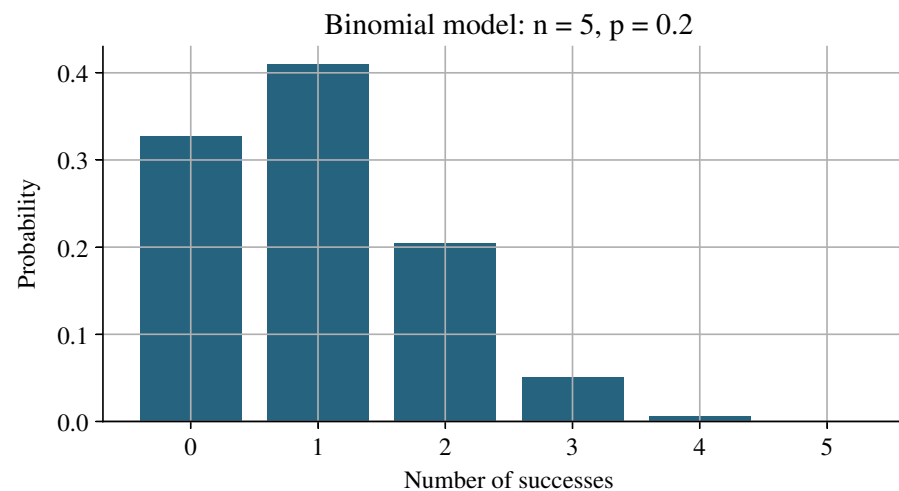
- A binomial model needs fixed n , independent trials, two outcomes and constant p .
- $E(X) = np$ and $\text{Var}(X) = np(1-p)$.
- For a normal model use $z = (x - \mu) / \sigma$.
- A Poisson model describes counts with a constant rate and appropriate independence assumptions.

■ A worked case

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad X \sim B(n, p)$$

Example:

For X binomial(5,0.2), $P(X=0)=0.8^5=0.32768$, $E(X)=1$ and $\text{Var}(X)=0.8$. For a normal quantity with $\mu=100, \sigma=15$, the value 130 has $z=2$. A Poisson mean of 3 per hour gives mean 6 over two hours.



$X \sim B(5, 0.2)$: $P(X = 1) \approx 0.41$ is the largest bar; $P(X = 5)$ is tiny

■ Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

Not true: Every count of successes has a binomial distribution regardless of dependence. The binomial model needs independent trials with a fixed probability; dependent trials break it.

■ Applying it

- Write the event as an inequality before using calculator distribution functions.
- Distinguish $P(X < k)$, $P(X \leq k)$ and a tail complement.
- State assumptions in context.

■ In your course

- Applications and Interpretation HL, the course first assessed in 2021.

Key:

- **Expected value:** The probability-weighted mean of a random variable.
- A binomial model needs fixed n , independent trials, two outcomes and constant p .

- Watch for: not every count is binomial: changing p or dependence can invalidate it.

■ Write from a z-score of 5

- A value 100 comes from a distribution with mean 85 and standard deviation 3.
- Find the z-score. A binomial count needs independent trials.

Hypothesis testing and contextual conclusions

■ Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
- This lesson studies **significance level** : The chosen probability threshold for rejecting a null hypothesis.

■ The method

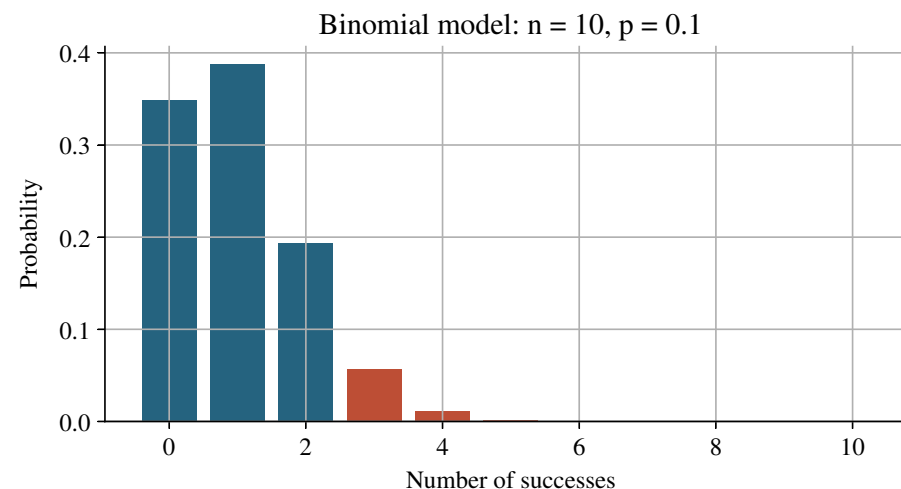
- State H_0 and H_1 in population parameters before inspecting the outcome.
- Calculate the appropriate tail probability under H_0 .
- Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.

■ A worked case

$$H_0 : p = p_0, \quad H_1 : p > p_0$$

Example:

For $H_0: p=0.1$ and $H_1: p>0.1$ with $n=10$, observing at least 3 defects has probability $1-(0.9^{10}+10 \times 0.1 \times 0.9^9 + 45 \times 0.1^2 \times 0.9^8) \approx 0.070191$. At 5%, this is insufficient evidence that the defect rate exceeds 10%.



Under $B(10, 0.1)$, $P(X \geq 3) \approx 0.07$: the shaded upper tail is the evidence region for the test

■ Where it goes wrong

- Failing to reject H_0 is not proof that H_0 is true.
- Choose the tail from H_1 , not from whichever tail gives a small result.
- Statistical significance does not measure the practical size of an effect.

Not true: Failing to reject a null hypothesis proves it is true. Not rejecting the null hypothesis only means there is not enough evidence against it.

■ Applying it

- Finish with a sentence about the population and the original claim.
- State the model's assumptions and consider whether the sampling procedure supports them.

■ In your course

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Key:

- **Significance level:** The chosen probability threshold for rejecting a null hypothesis.
- State H_0 and H_1 in population parameters before inspecting the outcome.
- Watch for: failing to reject H_0 is not proof that H_0 is true.

■ Write from a p-value of 0.12

- A one-tailed p-value is 0.12 and the significance level is 5 percent.
- Decide whether to reject the null hypothesis. Failing to reject does not prove it true.

Confidence intervals, chi-square and regression

■ How uncertain is the sample mean?

- Two samples give different means. We need a statement about sampling uncertainty, not a promise that one sample equals the population.
- This lesson studies **confidence interval** : An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate.

■ The method

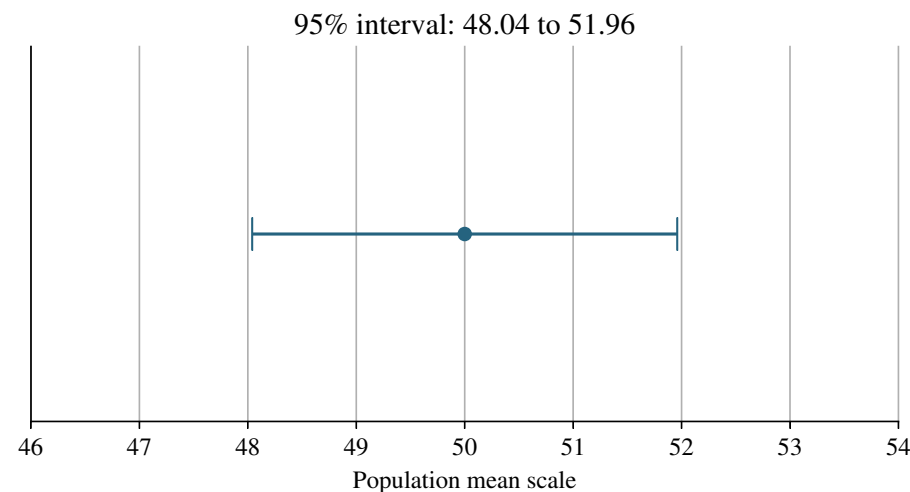
- For a normal mean with known σ , a 95% interval is sample mean $\pm 1.96\sigma/\sqrt{n}$.
- For chi-square, sum (observed-expected)²/expected with the correct degrees of freedom.
- Choose a test that fits the data and assumptions.

■ A worked case

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}, \quad \chi^2 = \sum \frac{(O - E)^2}{E}$$

Example:

With mean=50, $\sigma=10$, $n=100$, standard error=1 and the 95% interval is [48.04, 51.96]. For observed counts 30 and 20 against expected 25 and 25, chi-square=25/25+25/25=2, before deciding degrees of freedom and the rejection threshold.



$$95\% \text{ interval: } 50 \pm 1.96 \times 1 = 48.04 \text{ to } 51.96$$

■ Where it goes wrong

- A 95% confidence procedure does not give a 95% probability that a fixed population mean moves within this one interval.
- Expected counts and estimated parameters affect the validity and degrees of freedom of chi-square tests.

Not true: A strong correlation alone establishes a causal relationship. Correlation shows association; a third variable or chance can explain it, so causation needs an experiment or other evidence.

■ Applying it

- Report uncertainty in the context.
- In regression, inspect residuals, avoid unjustified extrapolation, and distinguish a fitted relationship from a causal explanation.
- S3 and IB AI HL require different verified scope boundaries.

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Key:

- **Confidence interval:** An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate.
- For a normal mean with known σ , a 95% interval is $\text{sample mean} \pm 1.96\sigma/\sqrt{n}$.
- Watch for: A 95% confidence procedure does not give a 95% probability that a fixed population mean moves within this one interval.

■ **Write from a chi-square value of 6**

- Observed counts are 18 and 6. Expected counts are 12 and 12.
- Find the chi-square statistic. A strong correlation does not prove a cause.