

Geometry and trigonometry

International Baccalaureate · IB Diploma · Mathematics:
Applications and Interpretation · HL

Scope and prerequisites

This handout covers selected HL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Angle reasoning, similarity and mensuration

■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

■ The method

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

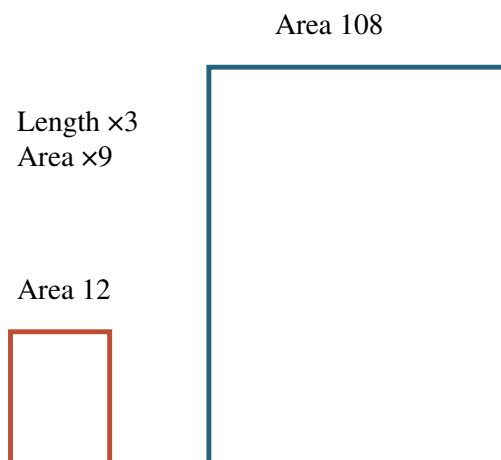
■ A worked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

Example:

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108 \text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216 \text{ cm}^3$. A cylinder with $r=3, h=5$ has volume $\pi r^2 h = 45\pi$.

A length scale factor is squared for area



Length scale factor 3 gives area scale factor $3^2 = 9$: $12 \times 9 = 108$

■ Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

Not true: Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

■ Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
 - A scale drawing is evidence only when the task permits measurement.
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■ In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- Watch for: equal angles alone establish similarity, not equal size.

■ Write from a volume factor of 216

- A length scale factor is 6.
- Find the area factor and the volume factor. Doubling lengths does not double the volume.

Right triangles and non-right triangles

■ Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
 - This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.
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■ The method

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite}/\text{hypotenuse}$, $\cos\theta = \text{adjacent}/\text{hypotenuse}$ and $\tan\theta = \text{opposite}/\text{adjacent}$.
 - For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.
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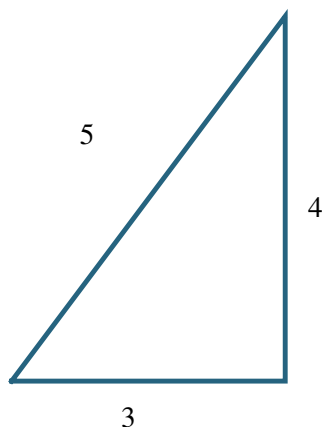
■ A worked case

$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Example:

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan\theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Label the sides first: here 3, 4 and hypotenuse 5, so $\sin = 4/5$, $\cos = 3/5$, $\tan = 4/3$ at the bottom-left angle

■ Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

■ Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
 - Explain why the chosen triangle contains the required length or angle.
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■ In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$.
- Watch for: label sides relative to the chosen angle.

■ Write from a hypotenuse of 29

- A right triangle has legs 20 and 21.
- Find the hypotenuse and the area. Pythagoras is only for right triangles.

Vectors and transformation geometry

■ Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
 - This lesson studies **resultant** : The vector sum representing the combined displacement or force.
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■ The method

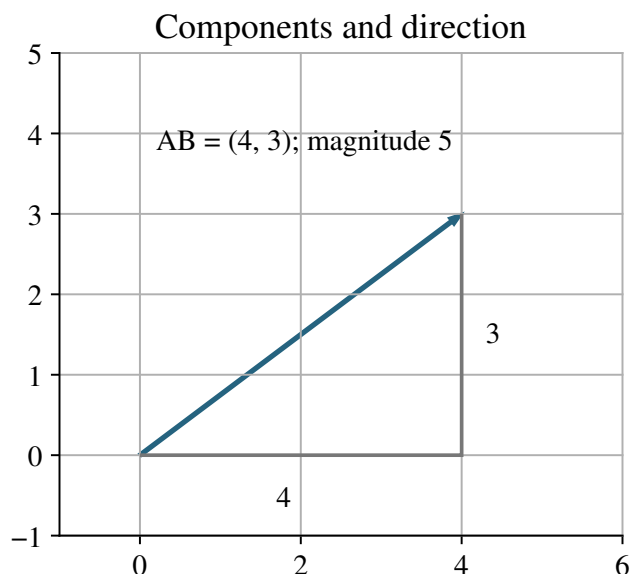
- Add corresponding vector components.
 - A displacement from A to B is $\overrightarrow{OB} - \overrightarrow{OA}$.
 - Parallel vectors are scalar multiples.
 - A vector has magnitude and direction; a distance is a scalar.
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■ A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

Example:

With $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 3)$, $\mathbf{a} + \mathbf{b} = (5, 4)$. Their scalar product is $\mathbf{a} \cdot \mathbf{b} = 4 \times 1 + 1 \times 3 = 7$. The magnitude of $\mathbf{a}$ is $\sqrt{17}$. For $\mathbf{A} = (1, 2)$, $\mathbf{B} = (5, 5)$, $\overrightarrow{AB} = (4, 3)$ and $|\overrightarrow{AB}| = 5$.



$$AB = (4, 3) \text{ has magnitude } \sqrt{4^2 + 3^2} = 5$$

■ Where it goes wrong

- The order of subtraction matters: $BA = -AB$.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

Not true: AB and BA always have the same components. BA goes the opposite way to AB , so $BA = -AB$: every component changes sign.

■ Applying it

- For a translation, every point uses the same displacement.
- For advanced line problems, distinguish the position vector from a direction vector and solve parameter equations consistently.

■ In your course

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Resultant:** The vector sum representing the combined displacement or force.
- Add corresponding vector components.

- Watch for: the order of subtraction matters: $\overrightarrow{BA} = -\overrightarrow{AB}$.

■ **Write from an x-component of 11**

- Add the vectors $(4, 2)$ and $(7, 1)$.
- Find the magnitude of $(5, 12)$. $\overrightarrow{AB}$ and $\overrightarrow{BA}$ do not have the same components.

■ **Write from a magnitude of 17**

- Find the magnitude of the vector $(-8, 15)$.
- $\sqrt{64 + 225} = 17$ – the components' Pythagoras.

■ **Write from a zero scalar product**

- Find the scalar product of $(3, -2)$ and $(4, 6)$.
- $12 - 12 = 0$ – perpendicular vectors, whatever their lengths.