Functions

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation · HL

Scope and prerequisites

This handout covers selected HL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Domains, inverses and composition

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

The method

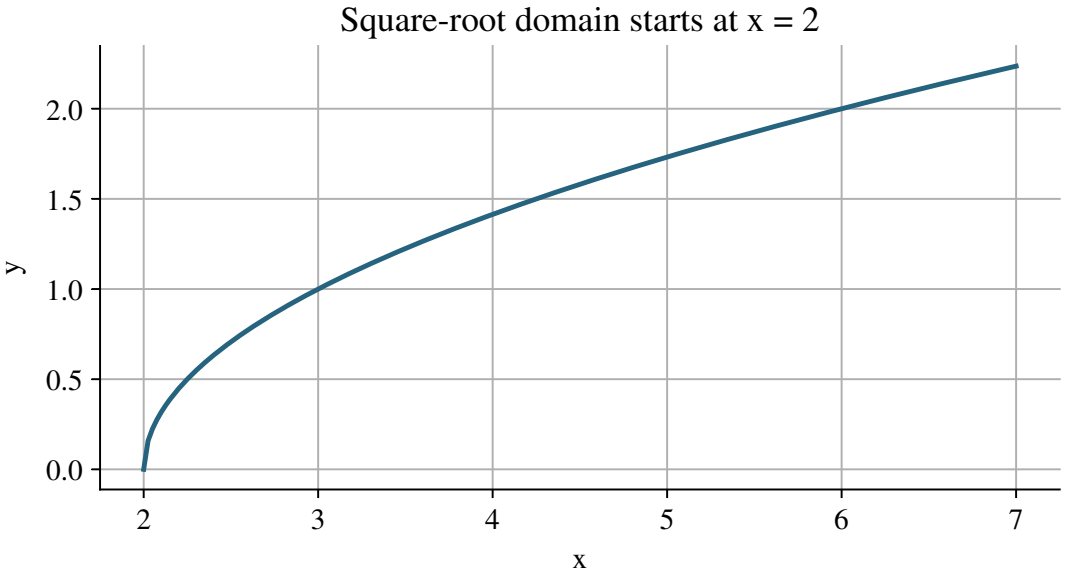
- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f; the intermediate output must be an allowed input to f.

A worked case

$$f(g(x)) = (f \circ g)(x), \qquad f^{-1}(f(x)) = x$$

Example:

For f(x)=√(x-2), x≥2 and the range is y≥0. From y=√(x-2), x=y^2+2. Thus f inverse(x)=x^2+2 with x≥0. For g(x)=x+3, fg(1)=f(4)=√2.



$y = \sqrt{(x - 2)}$ exists only for $x \geq 2$, so its domain starts at 2 and its range starts at 0

Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

Not true: Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one (f(1)=f(−1) for x^2), so its domain must be restricted first.

Applying it

- Check f(f inverse(x))=x on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Domain**: The set of allowed inputs to a function.

- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

■ Write from an inverse value of 8

- The function 8x plus 1 sends 8 to 65.
- A quadratic on all real numbers needs a restricted domain before it has an inverse.

Regression, financial models and residuals

■ Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
- This lesson studies **residual** : The observed value minus the value predicted by a fitted model.

■ The method

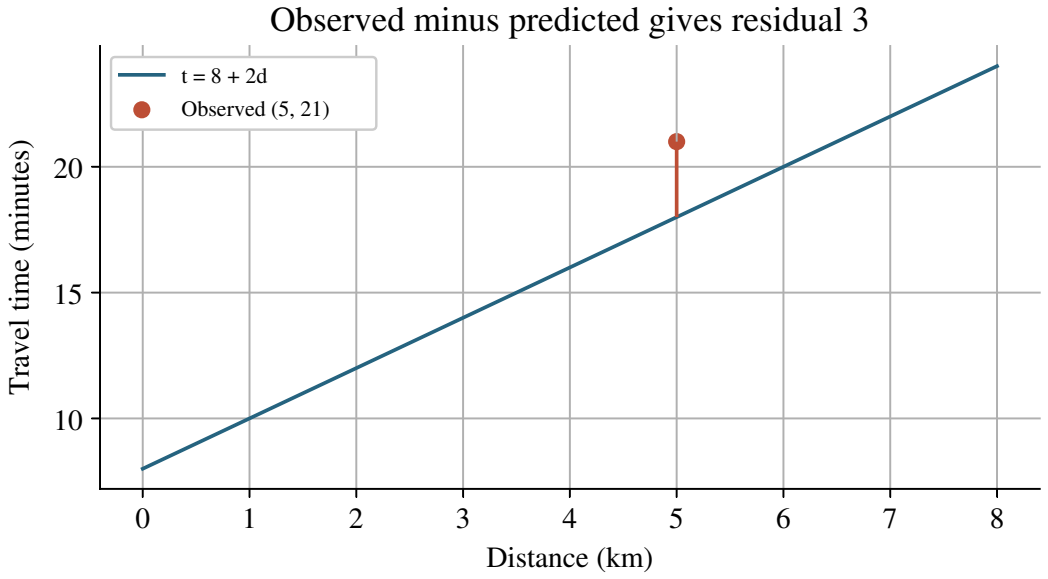
- A linear model y=a+bx has intercept a and slope b with contextual units.
- Inspect residuals and the data range.
- For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.

■ A worked case

$$\widehat{t} = 8 + 2d, \qquad e = t - \widehat{t}$$

Example:

The fitted model t=8+2d predicts 18 minutes at d=5 km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes 1000×1.05^3=1157.625 after 3 years.



Residual = observed − predicted = 21 − 18 = 3 minutes

Where it goes wrong

- A good fit does not prove a causal mechanism.
- A correlation coefficient measures linear association, not the gradient.
- Calculator output must be translated into a model, checked and interpreted.

Not true: A high correlation permits reliable extrapolation to any distance. A model fitted to data is reliable only near that data; extrapolating far outside it is unreliable.

Applying it

- In IB AI, record the data source, domain, assumptions and calculator method.
- Compare an alternative model and judge predictions against residuals.
- A financial answer must state payment timing and whether interest is compounded.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.

Key:

- **Residual**: The observed value minus the value predicted by a fitted model.
- A linear model y=a+bx has intercept a and slope b with contextual units.
- Watch for: A good fit does not prove a causal mechanism.

■ **Write from a prediction of 32**

- The model t = 2 + 6d predicts the time at d = 5.
- An observed time of 41 leaves a residual. A high correlation does not license extrapolation.

Exponentials, logarithms and modelling

■ **Why does a decay model stay positive?**

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.

■ **The method**

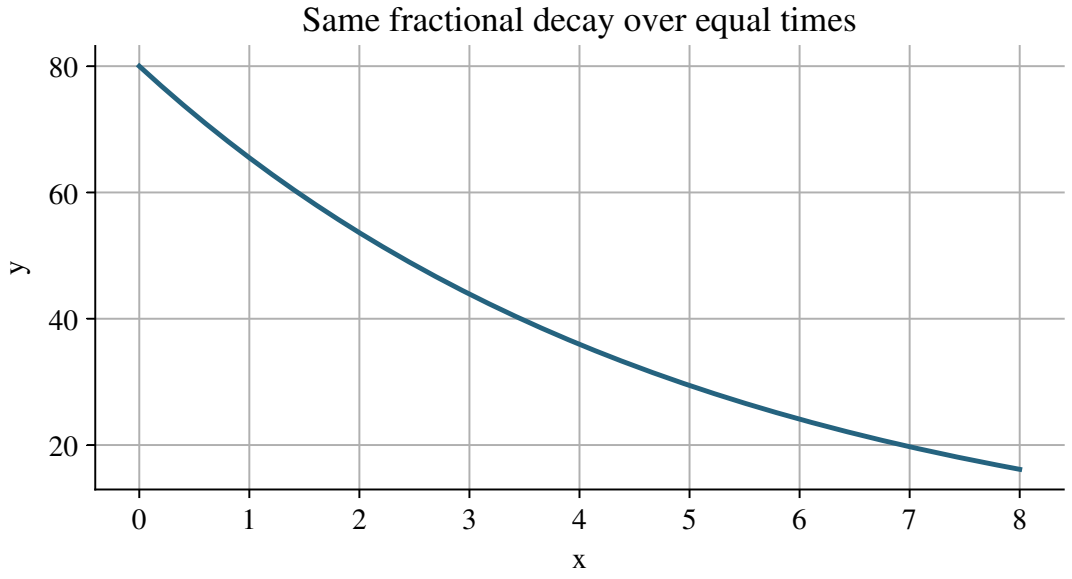
- For y=Ae^(kt), k is a proportional rate.
- Taking logs gives ln y=ln A+kt.
- Logarithms require positive arguments, and log(x+y) is not log x+log y.

■ **A worked case**

$$y = Ae^{kt}, \qquad \ln y = \ln A + kt$$

Example:

If y=80e^(-0.2t), y=40 gives e^(-0.2t)=1/2. Hence t=ln2/0.2≈3.466. For 3^x=20, x=ln20/ln3. Plotting ln y against t linearises this exponential model.



y = 80e^(−0.2x) loses the same fraction, about 18%, in every unit of x

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Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

Not true: For positive x and y, ln(x+y)=ln x+ln y. The log of a sum has no simple law; the product rule is ln(xy)=ln x+ln y.

Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Half-life**: The time for a decaying quantity to fall to half its initial value.
- For y=Ae^(kt), k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

Write from 2 to the power 9

- Solve 2 to the power x = 512.
- The sum of ln 5 and ln 4 is ln 20, not ln of the sum.

Write from a half-life of 10 days

- 200 g of a substance decays to 50 g in 20 days.
- Two halvings in 20 days, so the half-life is 10 days – count the halvings, not the subtraction.