5.4 Substitution, parts and partial fractions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS integration by parts

Objective

Substitution, parts and partial fractions. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** : An integration method based on the derivative of a product.

The method

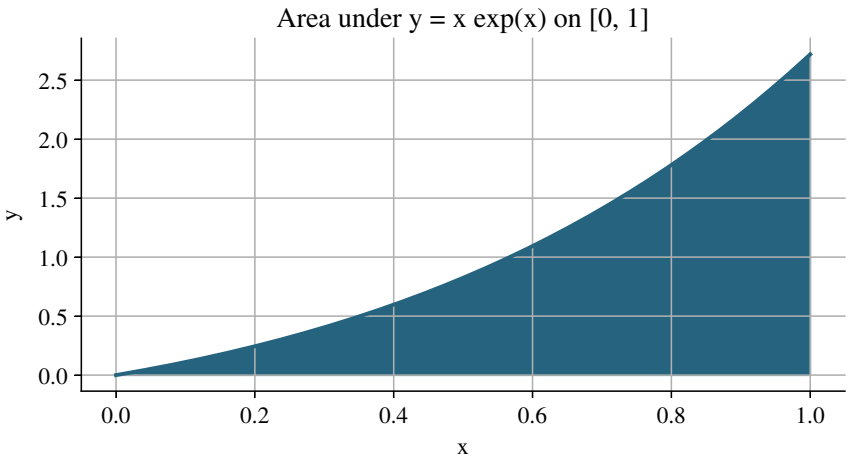
- Use substitution when an inner derivative appears as a factor.
- By parts, integral u v prime =uv-integral u prime v.
- For a rational function, divide first if needed, then decompose into partial fractions.

A worked case

$$\int uv'\,dx = uv - \int u'v\,dx$$

Example:

For integral xe^x dx, take u=x and v prime=e^x. Then the integral is xe^x-e^x+C. For integral 2x/(x²+1) dx, substitute w=x²+1, dw=2x dx; the answer is ln(x²+1)+C.



Where it goes wrong

- Choosing u and v prime well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

Not true: The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: ∫x · x dx=x³/3, not (x²/2)(x²/2).

Applying it

- Check by differentiating.
- For volumes of revolution around the x-axis use V=π integral y² dx; do not confuse the square of a function with the integral of the function.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Integration by parts**: An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

■ **Write from a volume factor of 20**

- The line y = 2 is revolved about the x-axis from 0 to 5. Find the volume divided by pi.
- The integral of a product is not the product of the integrals.

2 Practice

2.1 Evaluate the integral of xe^x from 0 to 1. Show the relation, substitution and units. [2]

2.2 Find the coefficient of ln(x²+1) in the integral of 2x/(x²+1). Show the relation, substitution and units. [2]

2.3 Find the volume divided by π when y=x is revolved about the x-axis from 0 to 3. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: The integral of a product equals the product of its separate integrals. [2]

2.5 Why does the substitution w=x²+1 fit the integral of 2x/(x²+1)? Choose one answer.

- **A** The numerator 2x is exactly the derivative of the inner expression, so it cancels
- **B** Because x²+1 has no antiderivative, a substitution is the only option
- **C** Substitution works for every quotient, whatever the numerator is [1]

3 Exam-style questions

3.1 Evaluate integral of x ln x from 1 to e by parts and differentiate the antiderivative as a check. [4]