# 4.2 Probability, trees and conditional reasoning

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

**KEY WORDS** conditional probability

## Objective

Probability, trees and conditional reasoning. Original classroom practice for IB HL; not an official IB paper.

## 1 Key knowledge and worked examples

### What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

### The method

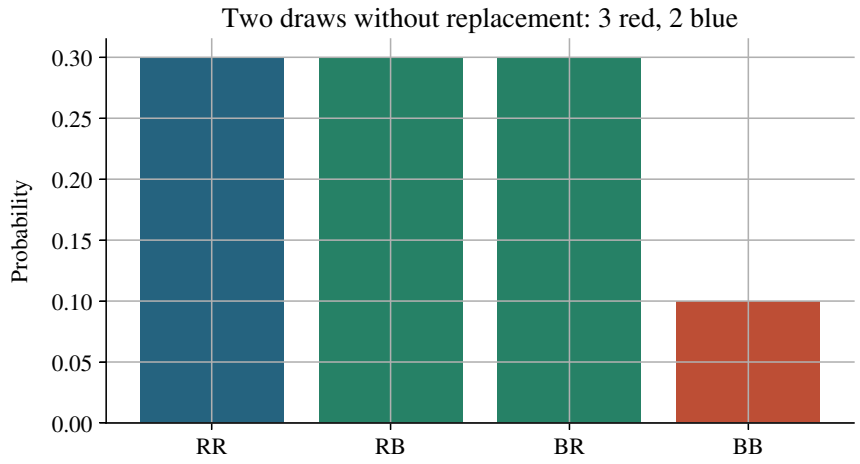
- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

### A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

**Example:**

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/5. If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.




### Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

**Not true:** Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so P(A and B)=0≠P(A)P(B).

### Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product P(A)P(B), justify independence from the context or the supplied information.

### In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

**Key:**

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

■ **Write from a conditional probability of 0.72**

- P(A and B) is 0.36 and P(B) is 0.50.
- Find P(A given B). Mutually exclusive events that can happen are not independent.

## 2 Practice

**2.1** Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

**2.2** Find P(A given B) if P(A∩B)=0.12 and P(B)=0.3. Show the relation, substitution and units. [2]

**2.3** Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

**2.4** Decide whether this claim is true and justify: Conditional probability means the probability of an event after restricting the sample space to a stated condition. [2]

**2.5** Two disjoint events A and B both have positive probability. Can they be independent? Choose one answer.

- **A** Yes: independence is automatic for all events
- **B** Yes: disjoint and independent mean the same thing
- **C** No: if A happens, P(B) drops to 0, so knowing A changes B [1]

## 3 Exam-style questions

**3.1** A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]