4.6 Hypothesis tests, errors and non-linear regression

Name: _____ Class: _____ Date: _____ **Total: 27 marks**

KEY WORDS hypothesis test • Type I error • Type II error • critical region • test statistic • coefficient of determination

Objective

Build the skills to answer exam questions on **hypothesis tests** for proportions, Poisson means and normal means, **Type I and Type II errors** , and **non-linear regression** .

You must be able to:

- test a binomial proportion or a Poisson mean using exact probabilities
- test a population mean with a z-test (σ known) or a t-test
- find critical regions and the probabilities of Type I and Type II errors
- compare non-linear regression models using the coefficient of determination R^2

1 Key knowledge and worked examples

■ Hypothesis tests, errors and non-linear regression

error	meaning	probability
Type I	reject H_0 when it is true	= actual significance level
Type II	do not reject H_0 when it is false	found from the true parameter

Exact tests: for $X \sim B(n,p)$ or $X \sim \text{Po}(m)$, the p-value is $P(X \text{ as extreme or more} \mid H_0)$, e.g. $P(X \geq 15)$ for $H_1 : p > 0.5$.

Mean, σ known: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$. **Mean, σ unknown:** t-test on the GDC.

Critical region: the set of values that lead to rejecting H_0; its total probability under H_0 must not exceed the significance level.

Non-linear regression: fit quadratic, cubic, exponential, power or sine models on the GDC; the model with R^2 closest to 1 fits the data best.

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Worked example. $H_0 : \mu = 50$, $\sigma = 6$, $n = 36$, $\bar{x} = 52$: $z = \dfrac{2}{1} = \mathbf{2}$.

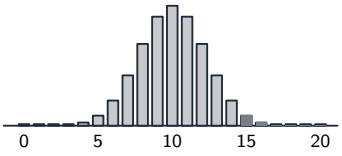
Answer shapes: a test always ends with a comparison (p against the level) and a sentence in context.

2 Practice

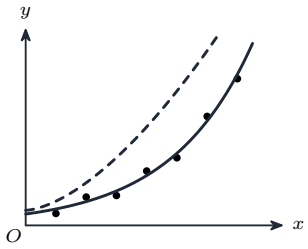
2.1 Explain what is meant by a Type I error and a Type II error. [2]

	H_0 true	H_0 false
reject	Type I	correct
do not reject	correct	Type II

2.2 A coin is tossed 20 times and shows 15 heads. Test at the 5% level whether the coin is biased towards heads. [3]



2.3 For one data set, an exponential model has $R^2 = 0.94$ and a quadratic model has $R^2 = 0.89$. Say what R^2 measures and which model fits better. [3]

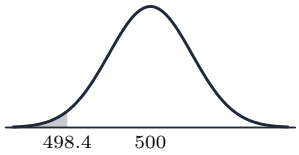


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3 Exam-style questions

3.1 A machine is set to fill bottles with a mean of 500 ml, with standard deviation 4 ml. A sample of 25 bottles has mean 498.4 ml. Test at the 5% level whether the machine is under-filling.

(a) State the hypotheses. [2]



(b) Find the test statistic. [2]

(c) Find the p-value. [2]

(d) State the conclusion. [1]

(e) Write down the probability of a Type I error, and explain what it means here. [2]

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3.2 A road had an average of 6 accidents a month. After new traffic lights, there were 2 accidents in one month. Test at the 5% level whether the rate has fallen.

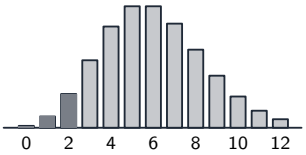
(a) State the hypotheses. [1]

(b) Find the p-value. [2]

(c) State the conclusion in context. [2]

(d) Find the critical region for this test. [2]

(e) The true rate is now 3 a month. Find the probability of a Type II error. [3]



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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

Type I: rejecting H_0 when it is true; Type II: not rejecting H_0 when it is false

2.2

$H_0 : p = 0.5$, $H_1 : p > 0.5$; $P(X \geq 15) = 0.0207$; $0.0207 < 0.05$: reject H_0, evidence of bias towards heads (R1)

2.3

R^2 is the proportion of the variation in y explained by the model; the **exponential** model, because 0.94 is closer to 1 (R1)

3.1

(a) $H_0 : \mu = 500$; $H_1 : \mu < 500$

(b) $z = \dfrac{498.4 - 500}{4/\sqrt{25}} = \mathbf{-2.0}$

(c) $P(Z < -2.0) = \mathbf{0.0228}$

(d) $0.0228 < 0.05$: reject H_0; evidence that the machine under-fills (R1)

(e) **0.05**; the chance of deciding the machine under-fills when it is in fact correct (R1)

3.2

(a) $H_0 : m = 6$, $H_1 : m < 6$

(b) $P(X \leq 2 \mid m = 6) = \mathbf{0.0620}$

(c) $0.0620 > 0.05$: do not reject H_0 (R1); not enough evidence that the lights reduced accidents

(d) $P(X \leq 1) = 0.0174$, $P(X \leq 2) = 0.0620$; $\mathbf{X \leq 1}$

(e) Type II: $X \geq 2$ when $m = 3$; $1 - P(X \leq 1 \mid m = 3) = \mathbf{0.801}$