4.4 Hypothesis testing and contextual conclusions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS significance level

Objective

Hypothesis testing and contextual conclusions. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
- This lesson studies **significance level** : The chosen probability threshold for rejecting a null hypothesis.

The method

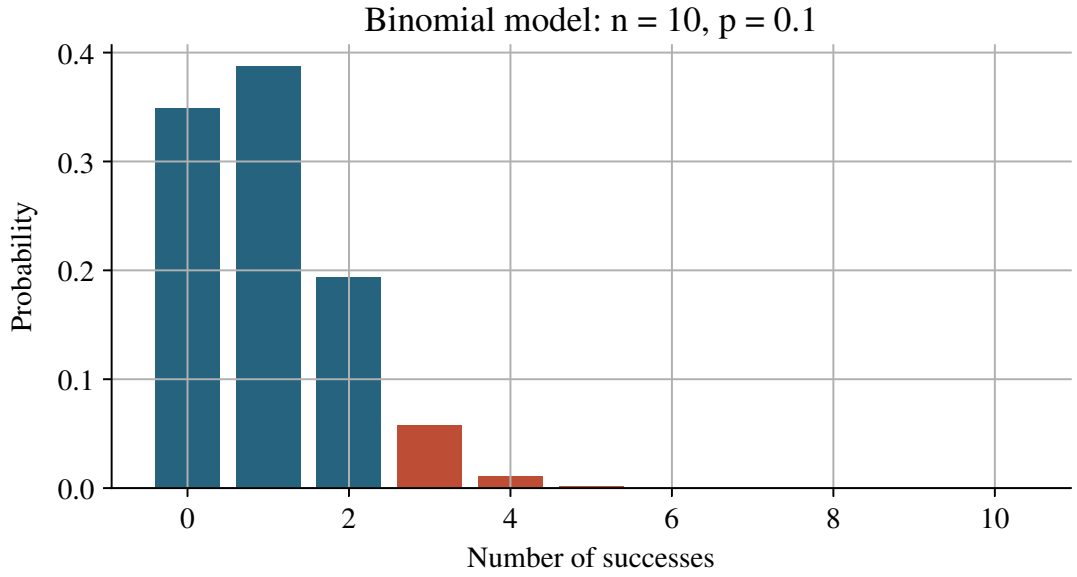
- State H_0 and H_1 in population parameters before inspecting the outcome.
- Calculate the appropriate tail probability under H_0.
- Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.

A worked case

$$H_0 : p = p_0, \qquad H_1 : p > p_0$$

Example:

For H_0:p=0.1 and H_1:p>0.1 with n=10, observing at least 3 defects has probability 1-(0.9^10+10×0.1×0.9^9+45×0.1^2×0.9^8)≈0.070191. At 5%, this is insufficient evidence that the defect rate exceeds 10%.



Where it goes wrong

- Failing to reject H_0 is not proof that H_0 is true.
- Choose the tail from H_1, not from whichever tail gives a small result.
- Statistical significance does not measure the practical size of an effect.

Not true: Failing to reject a null hypothesis proves it is true. Not rejecting the null hypothesis only means there is not enough evidence against it.

Applying it

- Finish with a sentence about the population and the original claim.
- State the model's assumptions and consider whether the sampling procedure supports them.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Significance level**: The chosen probability threshold for rejecting a null hypothesis.
- State H_0 and H_1 in population parameters before inspecting the outcome.
- Watch for: failing to reject H_0 is not proof that H_0 is true.

■ **Write from a p-value of 0.12**

- A one-tailed p-value is 0.12 and the significance level is 5 percent.
- Decide whether to reject the null hypothesis. Failing to reject does not prove it true.

2 Practice

2.1 Find the significance threshold written as a decimal for 5%. Show the relation, substitution and units. [2]

2.2 A correct one-tailed p-value is 0.03 at a 5% threshold. Enter 1 for reject or 0 for do not reject. Show the relation, substitution and units. [2]

2.3 Find P(X≥3) for X binomial(10,0.1). Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Significance level means the chosen probability threshold for rejecting a null hypothesis. [2]

2.5 At the 5% level the p-value is 0.07 for H_0:p=0.1. What is the correct conclusion? Choose one answer.

- **A** H_0 is proven true, because the test did not reject it
- **B** Insufficient evidence to reject H_0
- **C** H_1 is proven true, because 0.07 is close to 0.05 [1]

3 Exam-style questions

3.1 A correctly designed test gives p=0.04. State decisions at 5% and 1% significance and explain why p is not the probability H_0 is true. [4]

Solutions

2.1 5% means 5/100=0.05.

2.2 The correct tail probability 0.03 is below 0.05, so reject H_0. This is evidence, not proof.

2.3 Sum the binomial probabilities for k=3 through 10; equivalently use 1-P(X≤2)=0.0701908264.

2.4 True. True: The chosen probability threshold for rejecting a null hypothesis.

2.5 B 0.07 exceeds the 0.05 threshold, so the data do not meet the significance rule. That is not proof H_0 is true - the evidence is simply too weak against it.

3.1 Reject at 5%, do not reject at 1%. p is a tail probability under H_0 and the model, not a posterior probability. Do not change the chosen threshold after seeing the data.