3.4 Vectors, lines and motion

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

KEY WORDS vector • scalar product • vector product • velocity • closest approach • unit vector

Objective

Build the skills to answer exam questions on **vectors** : magnitude, scalar and vector products, lines, and motion with constant and changing velocity.

You must be able to:

- find magnitudes and unit vectors, and use the scalar product for angles
- use the vector product for perpendicular vectors and areas
- write vector equations of lines and test whether a point lies on a line
- model motion: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$, closest approach, and $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$

1 Key knowledge and worked examples

Vectors and motion

$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$. **Scalar product:** $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$. **Vector product:** $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$ = area of the parallelogram.

Line: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. **Constant velocity:** $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$; speed $= |\mathbf{v}|$.

question	method
do two objects collide?	same position at the **same** t
closest approach	write the distance $d(t)$ between them; minimize d^2
changing velocity	$\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$, $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$, component by component

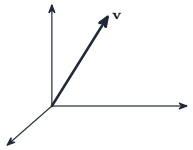
Worked example. $\mathbf{v} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ km h^{-1}: speed = **10** km h^{-1}.

Answer shapes: in a motion question, write the position of each object in terms of t first.

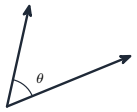
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2 Practice

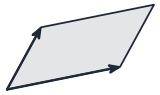
2.1 Find $|\mathbf{v}|$ and a unit vector in the direction of $\mathbf{v} = \begin{pmatrix} 3 \\ -4 \\ 12 \end{pmatrix}$. [2]



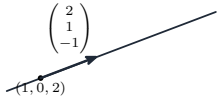
2.2 Find the angle between $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. [3]



2.3 Find $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ and the area of the parallelogram the two vectors make. [3]



2.4 The line L is $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$. Find the point where $\lambda = 3$, and show that $(5, 2, 0)$ lies on L. [3]



3 Exam-style questions

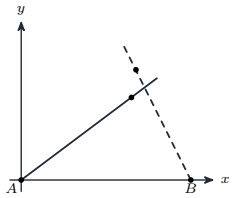
3.1 Boat A leaves $(0, 0)$ with velocity $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ km h^{-1}. At the same time boat B leaves $(10, 0)$ with velocity $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ km h^{-1}.

(a) Find the speed of boat A. [2]

(b) Find the position of boat B after 2 hours. [1]

(c) Show that the square of the distance between the boats after t hours is $(10 - 6t)^2 + t^2$. [2]

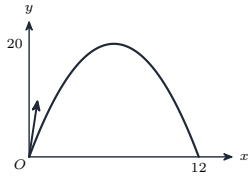
(d) Find the shortest distance between the boats. [3]



(e) Find when it happens. [1]

3.2 A ball's position after t seconds is $\mathbf{r} = \begin{pmatrix} 3t \\ 20t - 5t^2 \end{pmatrix}$ m.

(a) Find the velocity vector $\mathbf{v}$. [2]



(b) Find the speed at $t = 1$. [2]

(c) Find the greatest height. [2]

(d) Find the horizontal distance when the ball is back at height 0. [2]

(e) Find the angle between the starting velocity and the horizontal. [2]

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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

13; $\frac{1}{13}\begin{pmatrix}3\\-4\\12\end{pmatrix}$

2.2

$2-2+4=4$; $\cos\theta=\frac{4}{3\times3}$; $\theta=\mathbf{63.6}^\circ$

2.3

$\begin{pmatrix}6\\-3\\1\end{pmatrix}$; area $\sqrt{46}=\mathbf{6.78}$

2.4

$(\mathbf{7},\mathbf{3},-\mathbf{1})$; $\lambda=2$ from x; gives $y=2$, $z=0$, so it lies on L (R1)

3.1

(a) $\sqrt{4^2+3^2}=\mathbf{5}$ km h^{-1}

(b) $(\mathbf{6},\mathbf{8})$

(c) $A(4t,3t)$, $B(10-2t,4t)$; difference $\begin{pmatrix}10-6t\\t\end{pmatrix}$

(d) $d^2=37t^2-120t+100$; minimum at $t=\frac{120}{74}$; $d=\mathbf{1.64}$ km

(e) $t=\mathbf{1.62}$ hours

3.2

(a) $\mathbf{v}=\begin{pmatrix}3\\20-10t\end{pmatrix}$

(b) $\sqrt{3^2+10^2}=\mathbf{10.4}$ m s^{-1}

(c) $20-10t=0$, $t=2$; height **20** m

(d) $20t-5t^2=0$, $t=4$; **12** m

(e) $\tan^{-1}\frac{20}{3}=\mathbf{81.5}^\circ$

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