# 2.5 Composite functions, transformations, logistic models and linearizing data

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

**KEY WORDS** composite function • transformation • translation • stretch • logistic model • carrying capacity

## Objective

Build the skills to answer exam questions on **composite functions and transformations**, **logistic models** and **linearizing data with logarithms**.

**You must be able to:**

- form composite functions and inverse functions
- describe and apply translations, stretches and reflections of graphs
- use logistic models $f(x) = \dfrac{L}{1 + Ce^{-kx}}$ and interpret the carrying capacity $L$
- linearize $y = ab^x$ (semi-log) and $y = ax^n$ (log-log) and find $a$, $b$, $n$ from a line

## 1 Key knowledge and worked examples

### ■ Composites, transformations, logistic models and linearizing

| **transformation** | **effect on** $y = f(x)$ |
|---|---|
| $y = f(x) + d$ | translation by $\begin{pmatrix} 0 \\ d \end{pmatrix}$ |
| $y = f(x - h)$ | translation by $\begin{pmatrix} h \\ 0 \end{pmatrix}$ |
| $y = pf(x)$ | vertical stretch, scale factor $p$ |
| $y = f(qx)$ | horizontal stretch, scale factor $\frac{1}{q}$ |
| $y = -f(x)$, $y = f(-x)$ | reflection in the $x$-axis, in the $y$-axis |



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**Logistic model** $f(x) = \dfrac{L}{1 + Ce^{-kx}}$: growth slows down as $f$ approaches the **carrying capacity** $L$ (the horizontal asymptote).

**Linearizing:** if $y = ab^x$, then $\ln y = \ln a + x \ln b$: plot $\ln y$ against $x$. If $y = ax^n$, then $\ln y = \ln a + n \ln x$: plot $\ln y$ against $\ln x$; the gradient is $n$.

**Worked example.** $f(x) = x^2$, $g(x) = 3x - 1$: $(f \circ g)(x) = \mathbf{(3x - 1)^2}$.

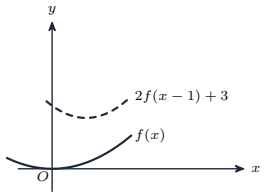
**Answer shapes:** describe each transformation with its type and its vector or scale factor.

## 2 Practice

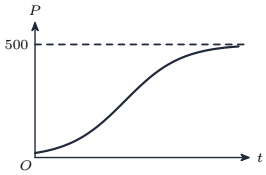
**2.1** $f(x) = x^2$ and $g(x) = x + 3$. Find $(f \circ g)(x)$ and $(g \circ f)(2)$. [2]



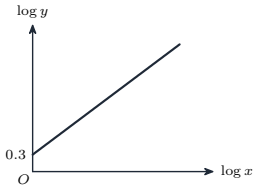

**2.2** Describe the transformations that map $y = f(x)$ onto $y = 2f(x - 1) + 3$. [3]




**2.3** $P = \dfrac{500}{1 + 24e^{-0.3t}}$. Find the initial value of $P$, the carrying capacity and $P$ when $t = 10$. [3]



**2.4** Data follow $y = ax^n$. The graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient 1.5 and intercept 0.3. Find $n$ and $a$. [3]



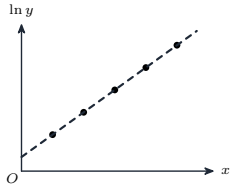

## 3 Exam-style questions

**3.1** The table shows data for a quantity $y$.

| $x$ | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| $y$ | 6 | 18 | 54 | 162 | 486 |

(a) Explain why a graph of $\ln y$ against $x$ is a straight line if $y = ab^x$. [2]

(b) Find the equation of the regression line of $\ln y$ on $x$. [3]




(c) Hence find $a$ and $b$. [2]

(d) Predict $y$ when $x = 6$. [2]

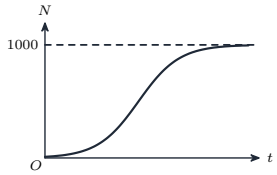


**3.2** The number of people who have heard a rumour $t$ days after it starts is $N(t) = \dfrac{1000}{1 + 99e^{-0.5t}}$.

(a) Find $N(0)$. [1]

(b) Write down the carrying capacity. [1]

(c) Find the time when 500 people have heard it. [3]




(d) Find $N(5)$. [2]

(e) Explain why a logistic model fits this situation better than an exponential model. [1]

(f) In another town the rumour starts 3 days later: $M(t) = N(t - 3)$. Describe the transformation and say what it means. [2]



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## Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

**2.1**

$(\mathbf{x+3})^{\mathbf{2}}$; $g(4) = \mathbf{7}$

**2.2**

translation $\begin{pmatrix}1\\0\end{pmatrix}$; vertical stretch, scale factor 2; translation $\begin{pmatrix}0\\3\end{pmatrix}$

**2.3**

**20**; **500**; **228**

**2.4**

$\log y = \log a + n \log x$; $n = \mathbf{1.5}$; $a = 10^{0.3} = \mathbf{2.00}$

**3.1**

(a) $\ln y = \ln a + x \ln b$, which has the form $Y = c + mx$ (R1)

(b) $\ln y$: $1.792, 2.890, 3.989, 5.088, 6.186$; $\ln y = \mathbf{1.10x + 0.693}$

(c) $a = e^{0.693} = \mathbf{2}$; $b = e^{1.0986} = \mathbf{3}$

(d) $2 \times 3^6 = \mathbf{1458}$

**3.2**

(a) **10**

(b) **1000**

(c) $1 + 99e^{-0.5t} = 2$; $e^{-0.5t} = \frac{1}{99}$; $t = 2\ln 99 = \mathbf{9.19}$ days

(d) $\dfrac{1000}{1 + 99e^{-2.5}} = \mathbf{110}$

(e) the number cannot grow for ever: it levels off at the size of the town (R1)

(f) translation $\begin{pmatrix}3\\0\end{pmatrix}$; the same spread happens 3 days later (R1)



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