2.5 Composite functions, transformations, logistic models and linearizing data

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

KEY WORDS composite function • transformation • translation • stretch • logistic model • carrying capacity

Objective

Build the skills to answer exam questions on **composite functions and transformations**, **logistic models** and **linearizing data with logarithms**.

You must be able to:

- form composite functions and inverse functions
- describe and apply translations, stretches and reflections of graphs
- use logistic models $f(x) = \dfrac{L}{1 + Ce^{-kx}}$ and interpret the carrying capacity L
- linearize $y = ab^x$ (semi-log) and $y = ax^n$ (log-log) and find a, b, n from a line

1 Key knowledge and worked examples

■ Composites, transformations, logistic models and linearizing

transformation	**effect on** $y = f(x)$
$y = f(x) + d$	translation by $\begin{pmatrix} 0 \\ d \end{pmatrix}$
$y = f(x - h)$	translation by $\begin{pmatrix} h \\ 0 \end{pmatrix}$
$y = pf(x)$	vertical stretch, scale factor p
$y = f(qx)$	horizontal stretch, scale factor $\frac{1}{q}$
$y = -f(x)$, $y = f(-x)$	reflection in the x-axis, in the y-axis

Logistic model $f(x) = \dfrac{L}{1 + Ce^{-kx}}$: growth slows down as f approaches the **carrying capacity** L (the horizontal asymptote).

Linearizing: if $y = ab^x$, then $\ln y = \ln a + x \ln b$: plot $\ln y$ against x. If $y = ax^n$, then $\ln y = \ln a + n \ln x$: plot $\ln y$ against $\ln x$; the gradient is n.

Worked example. $f(x) = x^2$, $g(x) = 3x - 1$: $(f \circ g)(x) = \mathbf{(3x - 1)^2}$.

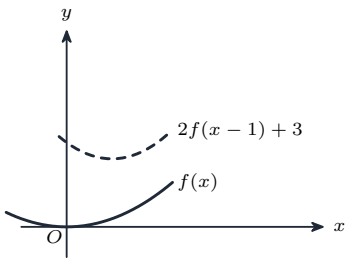
Answer shapes: describe each transformation with its type and its vector or scale factor.

2 Practice

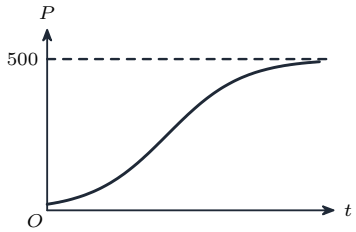
2.1 $f(x) = x^2$ and $g(x) = x + 3$. Find $(f \circ g)(x)$ and $(g \circ f)(2)$. [2]



2.2 Describe the transformations that map $y = f(x)$ onto $y = 2f(x - 1) + 3$. [3]

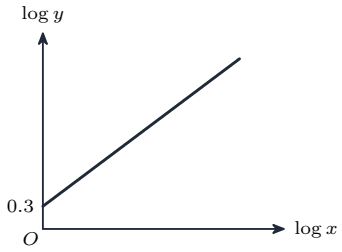


2.3 $P = \dfrac{500}{1 + 24e^{-0.3t}}$. Find the initial value of P, the carrying capacity and P when $t = 10$. [3]



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2.4 Data follow $y = ax^n$. The graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient 1.5 and intercept 0.3. Find n and a. [3]



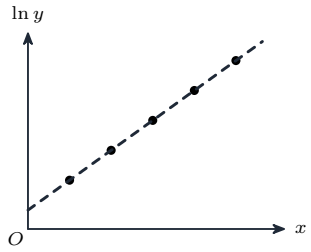
3 Exam-style questions

3.1 The table shows data for a quantity y.

x	**1**	**2**	**3**	**4**	**5**
y	6	18	54	162	486

(a) Explain why a graph of $\ln y$ against x is a straight line if $y = ab^x$. [2]

(b) Find the equation of the regression line of $\ln y$ on x. [3]



(c) Hence find a and b. [2]

(d) Predict y when $x = 6$. [2]

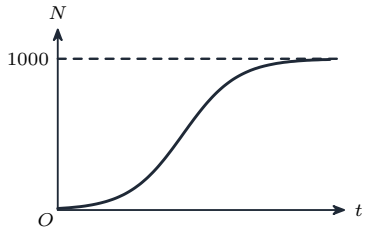
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3.2 The number of people who have heard a rumour t days after it starts is $N(t) = \frac{1000}{1 + 99e^{-0.5t}}$.

(a) Find $N(0)$. [1]

(b) Write down the carrying capacity. [1]

(c) Find the time when 500 people have heard it. [3]



(d) Find $N(5)$. [2]

(e) Explain why a logistic model fits this situation better than an exponential model. [1]

(f) In another town the rumour starts 3 days later: $M(t) = N(t - 3)$. Describe the transformation and say what it means. [2]

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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$(\mathbf{x}+\mathbf{3})^{\mathbf{2}}$; $g(4) = \mathbf{7}$

2.2

translation $\begin{pmatrix}1\\0\end{pmatrix}$; vertical stretch, scale factor 2; translation $\begin{pmatrix}0\\3\end{pmatrix}$

2.3

20; **500**; **228**

2.4

$\log y = \log a + n \log x$; $n = \mathbf{1.5}$; $a = 10^{0.3} = \mathbf{2.00}$

3.1

(a) $\ln y = \ln a + x \ln b$, which has the form $Y = c + mx$ (R1)

(b) $\ln y$: $1.792, 2.890, 3.989, 5.088, 6.186$; $\ln y = \mathbf{1.10x} + \mathbf{0.693}$

(c) $a = e^{0.693} = \mathbf{2}$; $b = e^{1.0986} = \mathbf{3}$

(d) $2 \times 3^6 = \mathbf{1458}$

3.2

(a) **10**

(b) **1000**

(c) $1 + 99e^{-0.5t} = 2$; $e^{-0.5t} = \frac{1}{99}$; $t = 2\ln 99 = \mathbf{9.19}$ days

(d) $\dfrac{1000}{1 + 99e^{-2.5}} = \mathbf{110}$

(e) the number cannot grow for ever: it levels off at the size of the town (R1)

(f) translation $\begin{pmatrix}3\\0\end{pmatrix}$; the same spread happens 3 days later (R1)

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