# 1.5 Matrix transformations, inverses and eigenvalues

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

**KEY WORDS** determinant

## Objective

Matrix transformations, inverses and eigenvalues. Original classroom practice for IB HL; not an official IB paper.

## 1 Key knowledge and worked examples

### Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** : A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

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### The method

- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Transform column vectors by left multiplication.
- The rightmost matrix acts first in a product.

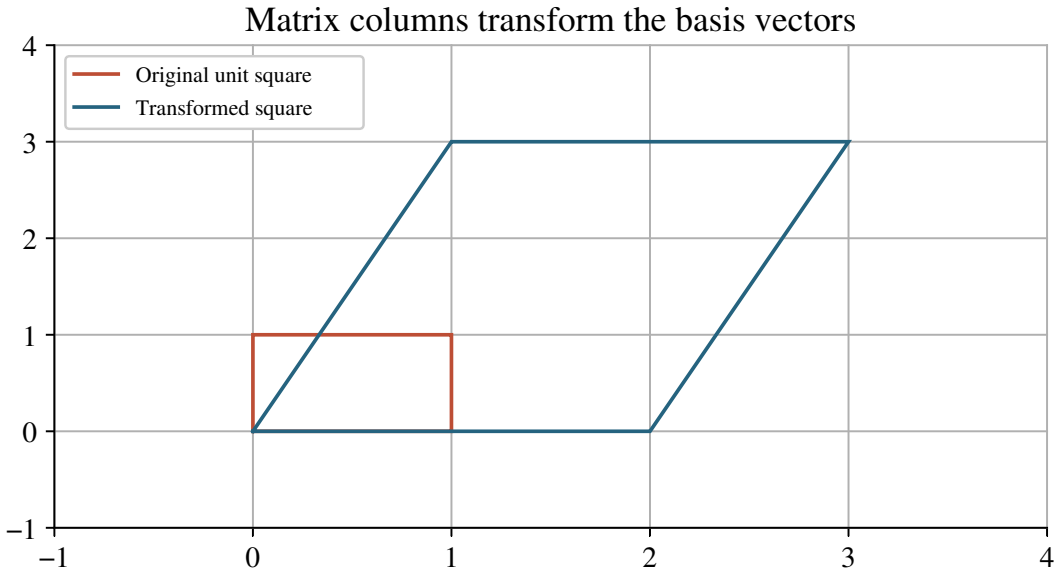
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### A worked case

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

**Example:**

For A=[[2,1],[0,3]], det A=6. A(1,2)=(4,6). Its inverse is [[1/2,-1/6],[0,1/3]]. The characteristic equation is (2-$\lambda$)(3-$\lambda$)=0, so the eigenvalues are 2 and 3.




## Where it goes wrong

- Matrix multiplication is usually not commutative.
- A determinant of zero means the transformation loses a dimension and has no inverse.
- Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

**Not true:** Every square matrix has an inverse. A square matrix with determinant 0 is singular and has no inverse.

## Applying it

- Find an eigenvector by solving $(A-\lambda I)v=0$ with $v\neq 0$.
- Explain the geometrical meaning: this vector keeps its line direction under the transformation.

## In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

**Key:**

- **Determinant**: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- A $2\times 2$ matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Watch for: matrix multiplication is usually not commutative.

■ **Write from a determinant of 8**

- The matrix has rows (4, 1) and (0, 2).
- Find its determinant. A square matrix with determinant 0 has no inverse.

## 2 Practice

**2.1** Find the determinant of [[2,1],[0,3]]. Show the relation, substitution and units. [2]

**2.2** Find the x-component after that matrix transforms (1,2). Show the relation, substitution and units. [2]

**2.3** Find the larger eigenvalue of that matrix. Show the relation, substitution and units. [2]

**2.4** Decide whether this claim is true and justify: Every square matrix has an inverse. [2]

**2.5** Why does a zero determinant mean a 2×2 matrix has no inverse? Choose one answer.

- **A** Only non-square matrices have inverses, and a zero determinant shows it is square
- **B** The inverse formula divides by ad-bc, and zero would mean squashing the plane onto a line
- **C** A matrix with a zero determinant has negative entries, which cannot be inverted [1]

## 3 Exam-style questions

**3.1** For A=[[2, 1], [0, 3]], find an eigenvector for each eigenvalue and check whether(1, 1) is an eigenvector. [4]

## Solutions

**2.1** The determinant is $2\times3-1\times0=6$.

**2.2** The first transformed coordinate is $2\times1+1\times2=4$.

**2.3** This triangular matrix has eigenvalues given by its diagonal entries 2 and 3, so the larger is 3.

**2.4** False. A square matrix with determinant 0 is singular and has no inverse.

**2.5 B** The determinant is the signed area scale factor. det=0 collapses a dimension - distinct points share an image - so no inverse transformation can separate them, and division by ad-bc fails.

**3.1** $\lambda$2:eigenvector(1, 0); $\lambda$3:(1, 1). A(1, 1)=(3, 3)=3(1, 1). Nonzero scalar multiples also work; the zero vector is excluded.