1.4 Repeated-deposit sequence models and timing

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS deposit

Objective

Repeated-deposit sequence models and timing. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Which timeline does the sum describe?

- Two equal payments can earn different amounts of interest if one is paid earlier.
- This lesson studies **deposit** : An amount added to an account at a stated time.

The method

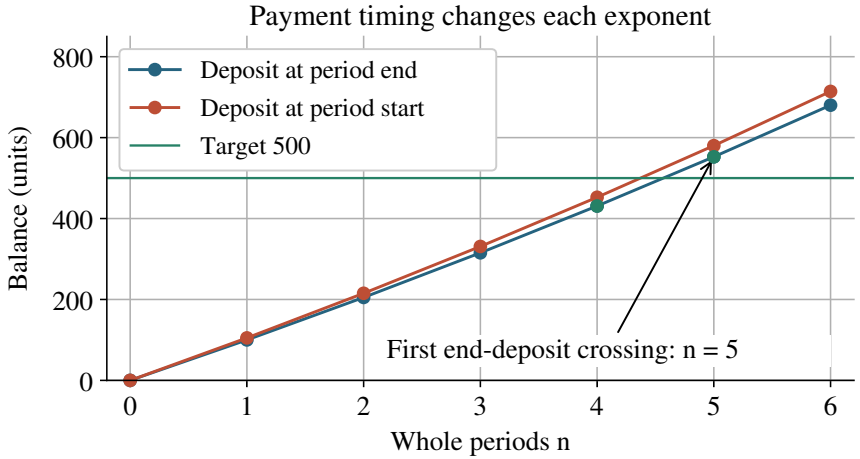
- Define the balance immediately after a stated event and give the initial balance.
- With growth factor r and an end-of-period deposit d, B_0=0 and B_n=rB_(n−1)+d.
- The nth balance is d(1+r+…+r^(n−1))=d(r^n−1)/(r−1) for r≠1.
- For r=1, use B_n=nd.
- The period count n is a nonnegative integer.

A worked case

$$B_0 = 0, \quad B_n = rB_{n-1} + d, \quad B_n = d\frac{r^n - 1}{r - 1} \ (r \neq 1)$$

Example:

Use an illustrative growth rate of 5% per period and deposit 100 units at each period end. B_1=100, B_2=1.05×100+100=205 and B_3=1.05×205+100=315.25. The first deposit earns interest for n−1 periods, while the final one earns none at the time B_n is measured. Thus B_n=100(1.05^n−1)/0.05. For a target of at least 500, B_4=431.0125<500 and B_5=552.563125≥500, so 5 periods first suffice. The balance exceeds the five deposits totalling 500 by 52.563125 units. If the same deposits are made at each period start and the balance is measured at the period end, C_n=1.05(B_n); hence C_3=331.0125, not 315.25.



Where it goes wrong

- Do not give the final end-of-period deposit an extra interest period.
- A balance includes both deposits and growth; it is not the interest alone.
- An exact target and an at-least target need different integer checks.
- A quoted annual rate cannot simply be used as a per-month factor without an appropriate conversion.

Not true: Beginning- and end-of-period deposits always give the same final balance. A deposit at the start of a period earns one more period of interest, so the final balance is larger.

Applying it

- The illustration assumes a fixed per-period rate, equal deposits, no fees or withdrawals and the stated compounding/payment times.
- If these change, refine the recurrence: B_n=(1+i_n)B_(n−1)+d_n−w_n for end-of-period deposits/withdrawals.
- Keep a separate ledger of payments to identify growth.
- Compare adjacent period balances before claiming the first target crossing; here all balances are nonnegative and B_n−B_(n−1)=0.05B_(n−1)+100>0.

■ **In your course**

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Deposit**: An amount added to an account at a stated time.
- Define the balance immediately after a stated event and give the initial balance.
- Watch for: do not give the final end-of-period deposit an extra interest period.

2 Practice

2.1 Find B_2 in the illustrative end-of-period model. Show the relation, substitution and units. [2]

2.2 Find B_3 in that model. Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.3 Find the first whole period with B_n≥500. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Deposit means an amount added to an account at a stated time. [2]

2.5 Why is the final end-of-period deposit not given an interest period in B_n? Choose one answer.

- **A** Every deposit earns n full periods of interest, including the last
- **B** Deposits never earn interest until the account is finally closed
- **C** The balance is measured at the moment it lands: the first deposit earns n-1 periods, the last earns none [1]

3 Exam-style questions

3.1 A student pays 200 into an account at the end of each month; it earns 1% per month and starts empty. Write a recurrence for the balance B_n just after the nth deposit, find B_3, and find the first month in which the balance reaches at least 1000. [4]

exercises.lol exercises.com.cn

Solutions

2.1 The second balance is 1.05×100+100=205.

2.2 The third balance is 1.05×205+100=315.25.

2.3 B_4=431.0125 is below 500 and B_5=552.563125 is above it; increasing balances give the first crossing at 5.

2.4 True. True: An amount added to an account at a stated time.

2.5 C With end-of-period deposits, B_n=d(1+r+...+r^(n-1)): the exponents count the periods EACH deposit has been sitting. Giving the last deposit interest would double-count time.

3.1 B_n=1.01B_{n-1}+200 with B_0=0, so B_1=200, B_2=402 and B_3=606.02. Summing the geometric series gives B_n=200(1.01^n−1)/0.01=20000(1.01^n−1). B_4=812.08<1000 and B_5=1020.20≥1000, so month 5 is the first. Paying at the start of each month instead makes every balance 1.01 times larger.

exercises.lol exercises.com.cn