1.4 Repeated-deposit sequence models and timing

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS deposit

Objective

Repeated-deposit sequence models and timing. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

Which timeline does the sum describe?

- Two equal payments can earn different amounts of interest if one is paid earlier.
- This lesson studies **deposit** : An amount added to an account at a stated time.

The method

- Define the balance immediately after a stated event and give the initial balance.
- With growth factor r and an end-of-period deposit d, B_0=0 and B_n=rB_(n−1)+d.
- The nth balance is d(1+r+…+r^(n−1))=d(r^n−1)/(r−1) for r≠1.
- For r=1, use B_n=nd.
- The period count n is a nonnegative integer.

A worked case

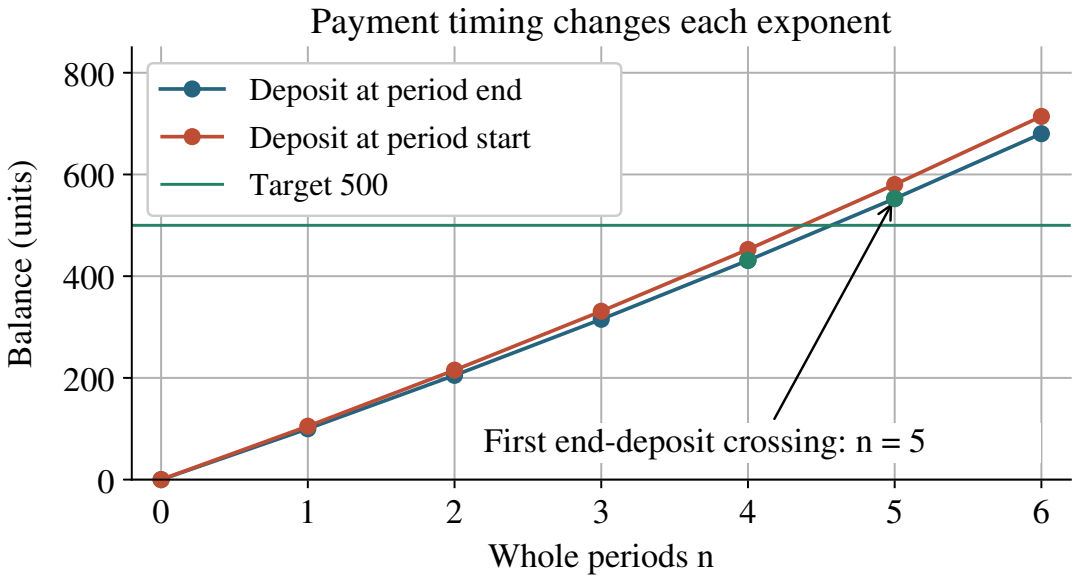
$$B_0 = 0, \quad B_n = rB_{n-1} + d, \quad B_n = d\frac{r^n - 1}{r - 1} \ (r \neq 1)$$

Example:

Use an illustrative growth rate of 5% per period and deposit 100 units at each period end. B_1=100, B_2=1.05×100+100=205 and B_3=1.05×205+100=315.25. The first deposit earns interest for n−1 periods, while the final one earns none at the time B_n is measured. Thus B_n=100(1.05^n−1)/0.05. For a target of at least 500, B_4=431.0125<500 and B_5=552.563125≥500, so 5 periods first suffice. The balance exceeds the five deposits totalling 500 by 52.563125 units. If the same deposits are made at each period start and

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the balance is measured at the period end, C_n=1.05(B_n); hence C_3=331.0125, not 315.25.



Where it goes wrong

- Do not give the final end-of-period deposit an extra interest period.
- A balance includes both deposits and growth; it is not the interest alone.
- An exact target and an at-least target need different integer checks.
- A quoted annual rate cannot simply be used as a per-month factor without an appropriate conversion.

Not true: Beginning- and end-of-period deposits always give the same final balance. A deposit at the start of a period earns one more period of interest, so the final balance is larger.

Applying it

- The illustration assumes a fixed per-period rate, equal deposits, no fees or withdrawals and the stated compounding/payment times.
- If these change, refine the recurrence: B_n=(1+i_n)B_(n−1)+d_n−w_n for end-of-period deposits/withdrawals.
- Keep a separate ledger of payments to identify growth.
- Compare adjacent period balances before claiming the first target crossing; here all balances are nonnegative and B_n−B_(n−1)=0.05B_(n−1)+100>0.

■ **In your course**

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Deposit**: An amount added to an account at a stated time.
- Define the balance immediately after a stated event and give the initial balance.
- Watch for: do not give the final end-of-period deposit an extra interest period.

2 Practice

2.1 Find B_2 in the illustrative end-of-period model. Show the relation, substitution and units. [2]

2.2 Find B_3 in that model. Give it to 3 significant figures. Show the relation, substitution and units. [2]

2.3 Find the first whole period with B_n≥500. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Deposit means an amount added to an account at a stated time. [2]

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2.5 Why is the final end-of-period deposit not given an interest period in B_n? Choose one answer.

- **A** Every deposit earns n full periods of interest, including the last
- **B** Deposits never earn interest until the account is finally closed
- **C** The balance is measured at the moment it lands: the first deposit earns n-1 periods, the last earns none [1]

3 Exam-style questions

3.1 A student pays 200 into an account at the end of each month; it earns 1% per month and starts empty. Write a recurrence for the balance B_n just after the nth deposit, find B_3, and find the first month in which the balance reaches at least 1000. [4]

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Solutions

2.1 The second balance is $1.05\times100+100=205$.

2.2 The third balance is $1.05\times205+100=315.25$.

2.3 $B_4=431.0125$ is below 500 and $B_5=552.563125$ is above it; increasing balances give the first crossing at 5.

2.4 True. True: An amount added to an account at a stated time.

2.5 C With end-of-period deposits, B_n=d(1+r+...+r^(n-1)): the exponents count the periods EACH deposit has been sitting. Giving the last deposit interest would double-count time.

3.1 $B_n=1.01B_{n-1}+200$ with $B_0=0$, so $B_1=200$, $B_2=402$ and $B_3=606.02$. Summing the geometric series gives $B_n=200(1.01^n-1)/0.01=20000(1.01^n-1)$. $B_4=812.08<1000$ and $B_5=1020.20\geq1000$, so month 5 is the first. Paying at the start of each month instead makes every balance 1.01 times larger.

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