Exercise walk-through

# 5.4
# Substitution, parts and partial fractions

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpret

# 5.4 Substitution, parts and partial fractions

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Name: _____ Class: _____ Date: _____ **Total: 13 marks**

**KEY WORDS** integration by parts

## Objective

Substitution, parts and partial fractions. Original classroom practice for IB HL; not an official IB paper.

## 1 Key knowledge and worked examples

### Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** : An integration method based

on the derivative of a product.

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## The method

- Use substitution when an inner derivative appears as a factor.
- By parts, integral u v prime =uv-integral u prime v.
- For a rational function, divide first if needed, then decompose into partial fractions.

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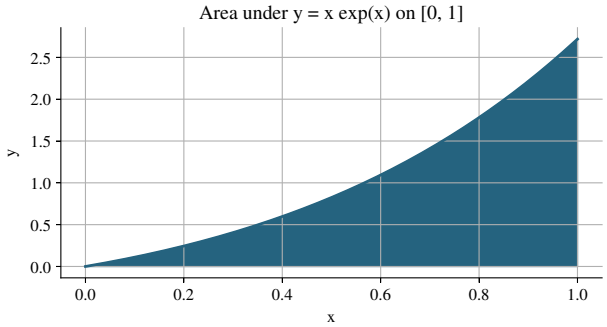
## A worked case

$$\int uv'\,dx = uv - \int u'v\,dx$$

**Example:**

For integral xe^x dx, take u=x and v prime=e^x. Then the integral is xe^x-e^x+C. For integral $2x/(x^2+1)$ dx, substitute $w=x^2+1$, $dw=2x$ dx; the answer is $\ln(x^2+1)+C$.

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■ **Where it goes wrong**

- Choosing u and v prime well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

**Not true:** The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: $\int x \cdot x \, dx = x^3/3$, not $(x^2/2)(x^2/2)$.

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■ **Applying it**

- Check by differentiating.
- For volumes of revolution around the x-axis use $V = \pi \int y^2 \, dx$; do not confuse the square of a function with the integral of the function.

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■ **In your course**

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

**Key:**

- **Integration by parts**: An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

## Write from a volume factor of 20

- The line y = 2 is revolved about the x-axis from 0 to 5. Find the volume divided by pi.
- The integral of a product is not the product of the integrals.

# Exercises 2.1·2.3

**2.1** Evaluate the integral of xe^x from 0 to 1. Show the relation, substitution and units. [2]

**2.2** Find the coefficient of $\ln(x^2+1)$ in the integral of $2x/(x^2+1)$. Show the relation, substitution and units. [2]

**2.3** Find the volume divided by $\pi$ when y=x is revolved about the x-axis from 0 to 3. Show the relation, substitution and units. [2]

# Exercises 2.1·2.3

**2.1** Evaluate the integral of xe^x from 0 to 1. Show the relation, substitution and units. [2]

**2.1** An antiderivative is (x-1)e^x. At 1 and 0 it gives 0 and -1, so the integral is 1.

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**2.2** Find the coefficient of $\ln(x^2+1)$ in the integral of $2x/(x^2+1)$. Show the relation, substitution and units. [2]

**2.2** Substitution $w=x^2+1$ gives $dw=2x\,dx$, so the coefficient of $\ln(x^2+1)$ is 1.

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**2.3** Find the volume divided by $\pi$ when $y=x$ is revolved about the x-axis from 0 to 3. Show the relation, substitution and units. [2]

**2.3** $V/\pi=\int x^2\,dx$ from 0 to $3=[x^3/3]=9$.

# Exercises 2.4·3.1

**2.4** Decide whether this claim is true and justify: The integral of a product equals the product of its separate integrals. [2]

**2.5** Why does the substitution $w=x^2+1$ fit the integral of $2x/(x^2+1)$? Choose one answer.

- **A** The numerator $2x$ is exactly the derivative of the inner expression, so it cancels
- **B** Because $x^2+1$ has no antiderivative, a substitution is the only option
- **C** Substitution works for every quotient, whatever the numerator is [1]

**3.1** Evaluate integral of x ln x from 1 to e by parts and differentiate the antiderivative as a check. [4]

# Exercises 2.4·3.1

**2.4** Decide whether this claim is true and justify: The integral of a product equals the product of its separate integrals. [2]

**2.4** False. A product needs a method such as integration by parts or substitution: $\int x \cdot x\,dx = x^3/3$, not $(x^2/2)(x^2/2)$.

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**2.5** Why does the substitution $w = x^2+1$ fit the integral of $2x/(x^2+1)$? Choose one answer.

- **A** The numerator $2x$ is exactly the derivative of the inner expression, so it cancels
- **B** Because $x^2+1$ has no antiderivative, a substitution is the only option
- **C** Substitution works for every quotient, whatever the numerator is [1]

**2.5 A** With $w = x^2+1$, $dw = 2x\,dx$, and the integral becomes $\int dw/w = \ln|w| + C$. Substitution is powerful precisely when the inner derivative appears as a factor.

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**3.1** Evaluate integral of x ln x from 1 to e by parts and differentiate the antiderivative as a check. [4]

**3.1** Take $u = \ln x$, $dv = x\,dx$: antiderivative $(x^2/2)\ln x - x^2/4$. Bounds give $(e^2+1)/4$. Differentiation returns $x \ln x$; domain $x > 0$ matters.