

Exercise walk-through

5.3

Chain, product, quotient and implicit differentiation

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation

5.3 Chain, product, quotient and implicit differentiation

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS chain rule

Objective

Chain, product, quotient and implicit differentiation. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

■ Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** : The rule that multiplies the outer derivative

by the inner derivative for a composite function.

■ The method

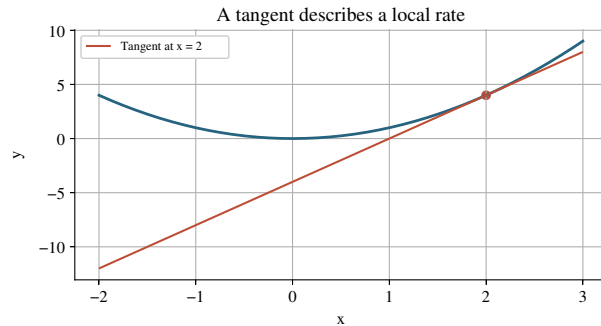
- For $y=f(g(x))$, $y' = f'(g(x))g'(x)$.
 - For uv , differentiate to $u'v + uv'$.
 - For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
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■ A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

Example:

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



■ Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

Not true: The derivative of $u(x)v(x)$ is always u prime times v prime. The product rule is $(uv)' = u'v + uv'$: for $x \cdot x$ the derivative is $2x$, not $1 \times 1 = 1$.

■ Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

■ In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Chain rule:** The rule that multiplies the outer derivative by the inner derivative for a composite function.
 - For $y = f(g(x))$, y prime $= f$ prime $(g(x))g$ prime (x) .
 - Watch for: A derivative of a product is not the product of the derivatives.
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■ Write from a product derivative of 17

- For $y = (3x \text{ plus } 2)(x \text{ plus } 1)$, find the derivative at $x = 2$.
- The derivative of a product is not the product of the derivatives.

Exercises 2.1·2.3

2.1 Find the derivative of $(3x+1)^4$ at $x=0$. Show the relation, substitution and units. [2]

2.2 For $x^2+y^2=25$, find dy/dx at $(3,4)$. Show the relation, substitution and units. [2]

2.3 For $y=xe^x$, find dy/dx at $x=0$. Show the relation, substitution and units. [2]

Exercises 2.1-2.3

2.1 Find the derivative of $(3x+1)^4$ at $x=0$. Show the relation, substitution and units. [2]

2.1 Chain rule gives $4(3x+1)^3 \times 3$; at zero this is 12.

2.2 For $x^2+y^2=25$, find dy/dx at $(3,4)$. Show the relation, substitution and units. [2]

2.2 Implicit differentiation gives $2x+2y y' = 0$, so $y' = -3/4$.

2.3 For $y=xe^x$, find dy/dx at $x=0$. Show the relation, substitution and units. [2]

2.3 Product rule gives $e^x + xe^x$; at zero this is 1.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: The derivative of $u(x)v(x)$ is always u prime times v prime. [2]

2.5 Why is the derivative of $x \cdot e^x$ not simply $1 \cdot e^x$? Choose one answer.

- **A** The product rule adds a second term: $u'v + uv'$ gives $e^x + xe^x$
- **B** Because e^x cannot be differentiated
- **C** The derivative of a product is the product of the derivatives [1]

3.1 For $x^2 + y^2 = 25$, find dy/dx and discuss its values at $(3, 4)$ and $(5, 0)$, using geometry to check. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: The derivative of $u(x)v(x)$ is always u prime times v prime. [2]

2.4 False. The product rule is $(uv)' = u'v + uv'$: for $x \cdot x$ the derivative is $2x$, not $1 \times 1 = 1$.

2.5 Why is the derivative of $x \cdot e^x$ not simply $1 \cdot e^x$? Choose one answer.

- **A** The product rule adds a second term: $u'v + uv'$ gives $e^x + xe^x$
- **B** Because e^x cannot be differentiated
- **C** The derivative of a product is the product of the derivatives [1]

2.5 A Each factor varies, so both contribute: with $u=x$ and $v=e^x$, the product rule gives $u'v + uv' = e^x + xe^x$. Multiplying the separate derivatives drops the uv' term.

3.1 For $x^2 + y^2 = 25$, find dy/dx and discuss its values at $(3, 4)$ and $(5, 0)$, using geometry to check. [4]

3.1 $y' = -x/y$ where $y \neq 0$. At $(3, 4)$, $-3/4$. At $(5, 0)$ the tangent is vertical, so finite dy/dx is undefined; implicit differentiation does not license division by zero.