

Exercise walk-through

3.3

Vectors and transformation geometry

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation

3.3 Vectors and transformation geometry

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS resultant

Objective

Vectors and transformation geometry. Original classroom practice for IB HL; not an official IB paper.

1 Key knowledge and worked examples

■ Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** : The vector sum representing the combined

displacement or force.

■ The method

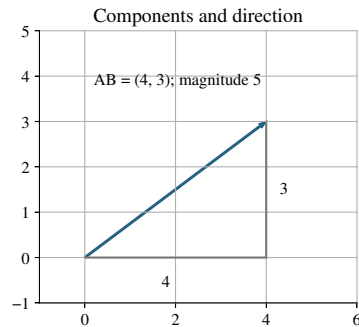
- Add corresponding vector components.
 - A displacement from A to B is $\mathbf{OB} - \mathbf{OA}$.
 - Parallel vectors are scalar multiples.
 - A vector has magnitude and direction; a distance is a scalar.
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■ A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

Example:

With $\mathbf{a}=(4,1)$ and $\mathbf{b}=(1,3)$, $\mathbf{a}+\mathbf{b}=(5,4)$. Their scalar product is $\mathbf{a} \cdot \mathbf{b}=4 \times 1+1 \times 3=7$. The magnitude of $\mathbf{a}$ is $\sqrt{17}$. For $\mathbf{A}=(1,2)$, $\mathbf{B}=(5,5)$, $\mathbf{AB}=(4,3)$ and $|\mathbf{AB}|=5$.



■ **Where it goes wrong**

- The order of subtraction matters: $\mathbf{BA} = -\mathbf{AB}$.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

Not true: $\mathbf{AB}$ and $\mathbf{BA}$ always have the same components. $\mathbf{BA}$ goes the opposite way to $\mathbf{AB}$, so $\mathbf{BA} = -\mathbf{AB}$: every component changes sign.

■ **Applying it**

- For a translation, every point uses the same displacement.
 - For advanced line problems, distinguish the position vector from a direction vector and solve parameter equations consistently.
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■ **In your course**

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key:

- **Resultant:** The vector sum representing the combined displacement or force.
 - Add corresponding vector components.
 - Watch for: the order of subtraction matters: $\mathbf{BA} = -\mathbf{AB}$.
-

■ Write from an x-component of 11

- Add the vectors $(4, 2)$ and $(7, 1)$.
- Find the magnitude of $(5, 12)$. AB and BA do not have the same components.

Exercises 2.1·2.3

2.1 Find the x-component of $(4,1)+(1,3)$. Show the relation, substitution and units. [2]

2.2 Find the magnitude of $(4,3)$. Show the relation, substitution and units. [2]

2.3 Find the scalar product of $(4,1)$ and $(1,3)$. Show the relation, substitution and units. [2]

Exercises 2.1-2.3

2.1 Find the x-component of $(4,1)+(1,3)$. Show the relation, substitution and units. [2]

2.1 Add the x-components: $4+1=5$.

2.2 Find the magnitude of $(4,3)$. Show the relation, substitution and units. [2]

2.2 Magnitude= $\sqrt{4^2+3^2}=5$.

2.3 Find the scalar product of $(4,1)$ and $(1,3)$. Show the relation, substitution and units. [2]

2.3 Multiply corresponding components and add: $4 \times 1 + 1 \times 3 = 7$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Resultant means the vector sum representing the combined displacement or force. [2]

2.5 What is the displacement vector from $A=(1,2)$ to $B=(5,5)$? Choose one answer.

- **A** (6,7): add the two position vectors to get the displacement
- **B** (4,3): subtract the start position from the end position
- **C** (3,4): subtract the start from the end and then swap the components [1]

3.1 Points $A=(1, 2, 0)$, $B=(3, 2, 1)$, $C=(1, 5, 0)$. Find $\overrightarrow{AB} \cdot \overrightarrow{AC}$ and decide whether the angle at A is right; distinguish a zero vector exception. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Resultant means the vector sum representing the combined displacement or force. [2]

2.4 True. True: The vector sum representing the combined displacement or force.

2.5 What is the displacement vector from $A=(1,2)$ to $B=(5,5)$? Choose one answer.

- **A** (6,7): add the two position vectors to get the displacement
- **B** (4,3): subtract the start position from the end position
- **C** (3,4): subtract the start from the end and then swap the components [1]

2.5 B $AB = OB - OA = (5,5)-(1,2) = (4,3)$. The order matters: BA would be $(-4,-3)$, the opposite direction.

3.1 Points $A=(1, 2, 0)$, $B=(3, 2, 1)$, $C=(1, 5, 0)$. Find $AB \cdot AC$ and decide whether the angle at A is right; distinguish a zero vector exception. [4]

3.1 $AB=(2, 0, 1)$, $AC=(0, 3, 0)$, dot 0. Both nonzero, so angle $\pi/2$. A zero dot product involving a zero vector would not define an angle.