

Exercise walk-through

# 3.1

## Angle reasoning, similarity and mensuration

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International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation

# 3.1 Angle reasoning, similarity and mensuration

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 13 marks**

**KEY WORDS** scale factor

# Objective

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Angle reasoning, similarity and mensuration. Original classroom practice for IB HL; not an official IB paper.

## 1 Key knowledge and worked examples

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### ■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding

lengths in similar shapes.

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## ■ The method

- For similar shapes with length scale factor  $k$ , areas scale by  $k^2$  and volumes by  $k^3$ .
  - State angle reasons explicitly.
  - A circle's tangent is perpendicular to the radius at the contact point.
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## ■ A worked case

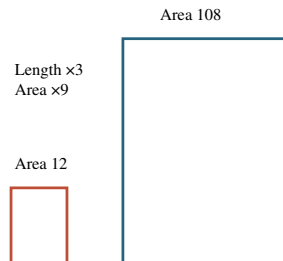
$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

**Example:**

If model-to-real length factor is 3, a model area of  $12 \text{ cm}^2$  gives  $12 \times 3^2 = 108 \text{ cm}^2$  and a model volume of  $8 \text{ cm}^3$  gives  $8 \times 3^3 = 216 \text{ cm}^3$ . A cylinder with  $r=3, h=5$  has volume  $\pi r^2 h = 45\pi$ .

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A length scale factor is squared for area



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■ **Where it goes wrong**

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

**Not true:** Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

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■ **Applying it**

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

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■ **In your course**

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

**Key:**

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.
  - For similar shapes with length scale factor  $k$ , areas scale by  $k^2$  and volumes by  $k^3$ .
  - Watch for: equal angles alone establish similarity, not equal size.
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■ **Write from a volume factor of 216**

- A length scale factor is 6.
- Find the area factor and the volume factor. Doubling lengths does not double the volume.

## Exercises 2.1-2.3

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**2.1** Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

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**2.2** Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

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**2.3** Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

## Exercises 2.1-2.3

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**2.1** Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

**2.1** Area scales by the square of the length factor:  $3^2=9$ .

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**2.2** Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

**2.2** Volume scales by the cube:  $3^3=27$ .

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**2.3** Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

**2.3** Interior-angle sum  $= (5-2) \times 180 = 540^\circ$ .

## Exercises 2.4·3.1

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**2.4** Decide whether this claim is true and justify: Scale factor means the multiplier that relates corresponding lengths in similar shapes. [2]

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**2.5** Two similar solids have length scale factor 3. Why is the volume factor 27? Choose one answer.

- **A** Volume scales with the length factor itself: a factor of 3 gives 3
- **B** Volume scales with the square of the length factor:  $3^2=9$ , then add the base
- **C** Volume scales with the cube of the length factor:  $3^3=27$  [1]

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**3.1** Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

## Exercises 2.4·3.1

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**2.4** Decide whether this claim is true and justify: Scale factor means the multiplier that relates corresponding lengths in similar shapes. [2]

**2.4** True. True: The multiplier that relates corresponding lengths in similar shapes.

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**2.5** Two similar solids have length scale factor 3. Why is the volume factor 27? Choose one answer.

- **A** Volume scales with the length factor itself: a factor of 3 gives 3
- **B** Volume scales with the square of the length factor:  $3^2=9$ , then add the base
- **C** Volume scales with the cube of the length factor:  $3^3=27$  [1]

**2.5 C** Each of the three dimensions multiplies by 3, so volume multiplies by  $3 \times 3 \times 3 = 27$ . Area uses the square and volume the cube of the length factor.

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**3.1** Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

**3.1** Length ratio 3:2, area 9:4. Area scales as the square and volume as the cube of the length factor, assuming genuine geometric similarity.