Course notes

International Baccalaureate · IB Diploma · Mathematics: Applications and Interpretation · HL

AI: course teaching notes

Version: First assessment 2021

These are original teaching notes, and each lesson is labelled with its tier.

1 · Exact arithmetic and estimation

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** 质因数: A prime number that divides the integer exactly.

The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

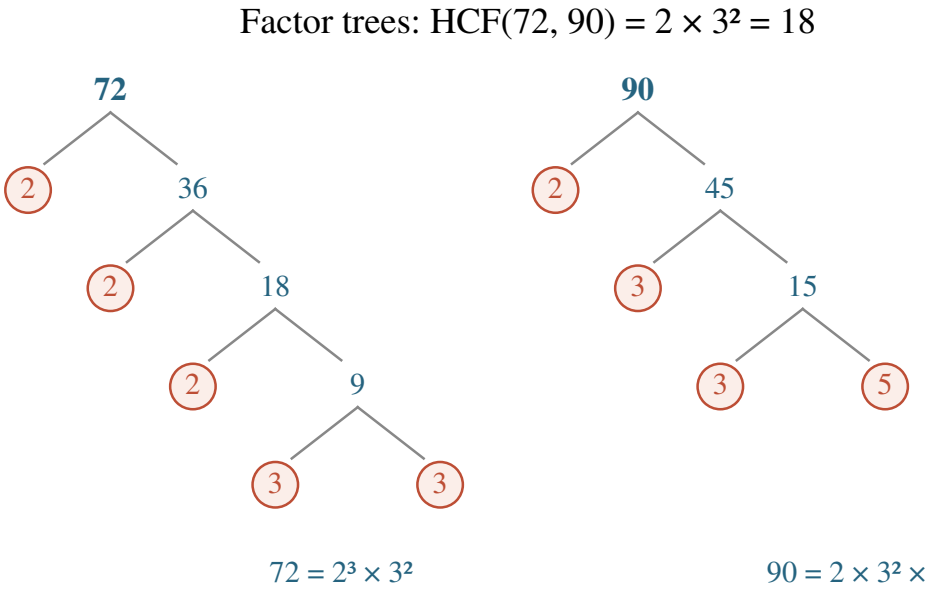
A worked case

$$a=\prod p_i^{\alpha_i},\quad b=\prod p_i^{\beta_i},\quad \mathrm{HCF}(a,b)=\prod p_i^{\min(\alpha_i,\beta_i)}$$

■ Worked example

72=2^3×3^2 and 90=2×3^2×5. Their HCF is 2×9=18. Make 18 bags with 4 pencils and 5 pens each. Their LCM is 2^3×3^2×5=360.

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Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

> ■ **Watch out**
>
> **Not true:** The HCF of two positive integers is larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.

■ **Key idea**

- **Prime factor**: A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

1 · Percentages, ratio and proportional reasoning

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier** 乘数: A factor that performs a percentage change in one multiplication.

The method

- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Reverse a percentage by dividing by the multiplier.
- In a ratio, first find the total number of parts.

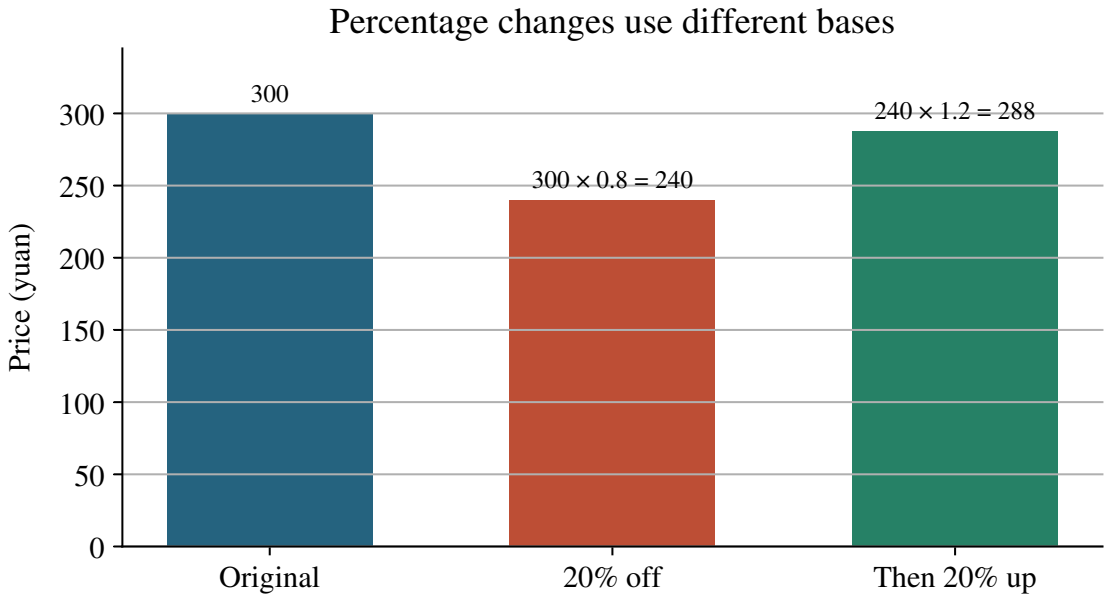
A worked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

■ **Worked example**

Let the original price be P. The model is sale price=0.8P. Hence P=240/0.8=300. A later 20% increase gives 240×1.2=288, so the two changes do not cancel.

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Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

> **Watch out**
>
> **Not true:** A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100\times0.8\times1.2=96$, not 100.

Applying it

- For direct proportion use y=kx; for inverse proportion use y=k/x.
- Calculate k from a known pair before using a new value.
- State what you held constant.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Multiplier**: A factor that performs a percentage change in one multiplication.
- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Watch for: A percentage uses a stated base.

1 · Sequences, series and recurrence

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** 公比: The constant multiplier between consecutive terms of a geometric sequence.

The method

- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- For a geometric progression, u_n=ar^(n-1) and S_n=a(1-r^n)/(1-r).
- An infinite geometric sum exists only if |r|<1.

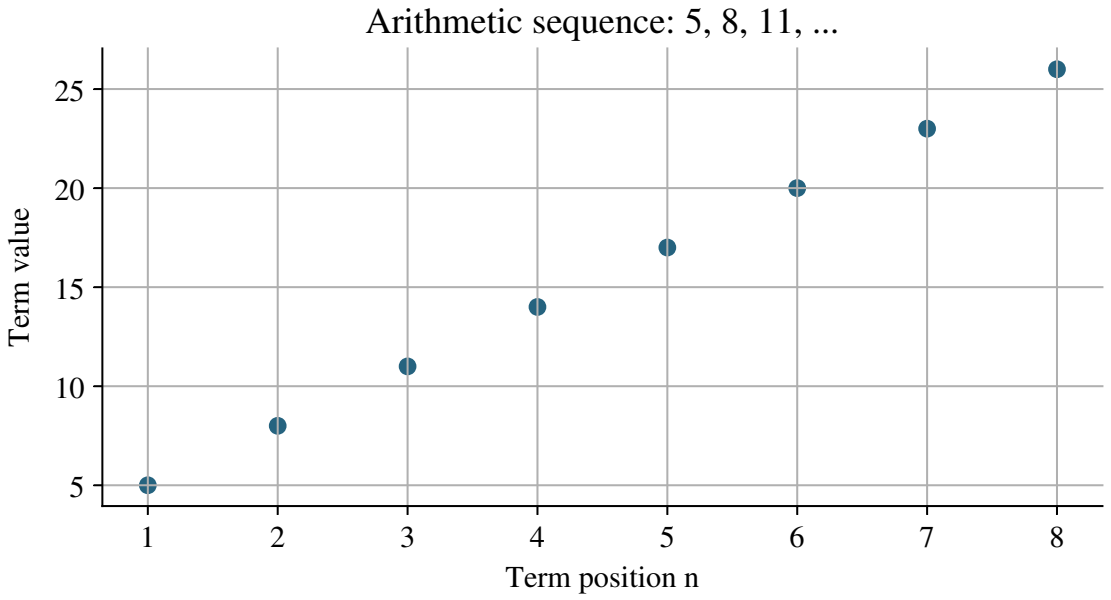
A worked case

$$u_n = a + (n-1)d, \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Worked example

For a=5,d=3,n=8, u_8=5+7×3=26 and S_8=8(10+21)/2=124. For a=12,r=1/2, S infinity=12/(1-1/2)=24. For u_(n+1)=2u_n+1 with u_1=1, the next terms are 3,7,15.

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Where it goes wrong

- The first term has index 1, so the exponent is n-1.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

> ■ **Watch out**
>
> **Not true:** Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1<r<1$; for $r=2$ the terms grow without limit.

Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Common ratio**: The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- Watch for: the first term has index 1, so the exponent is n-1.

1 · Repeated-deposit sequence models and timing

Which timeline does the sum describe?

- Two equal payments can earn different amounts of interest if one is paid earlier.
- This lesson studies **deposit** 存款: An amount added to an account at a stated time.

The method

- Define the balance immediately after a stated event and give the initial balance.
- With growth factor r and an end-of-period deposit d, $B_0=0$ and B_n=rB_(n−1)+d.
- The nth balance is d(1+r+…+r^(n−1))=d(r^n−1)/(r−1) for r≠1.
- For r=1, use B_n=nd.
- The period count n is a nonnegative integer.

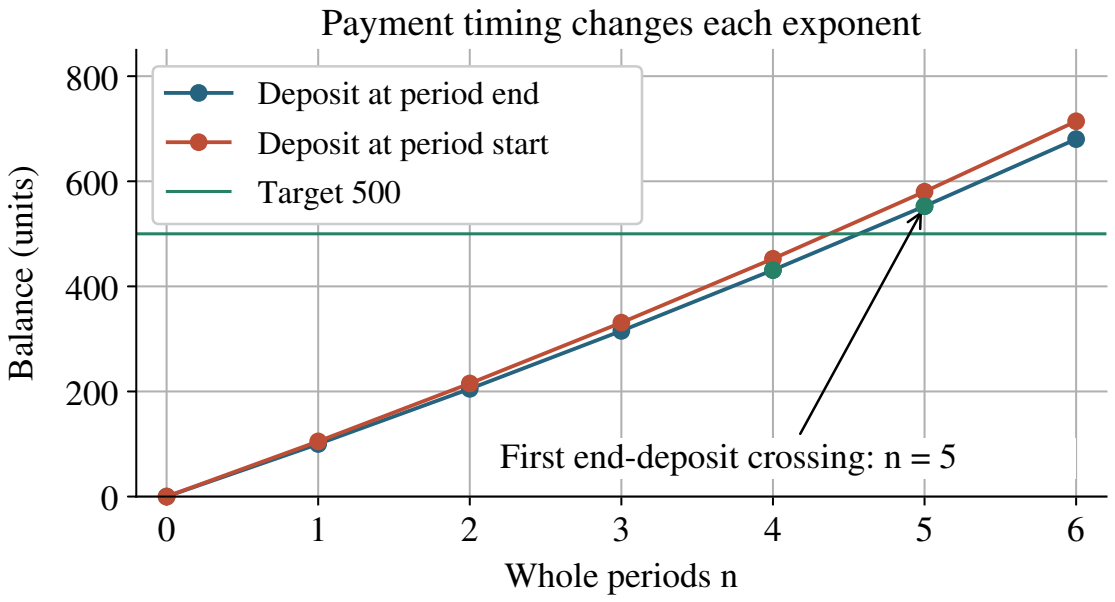
A worked case

$$B_0 = 0, \quad B_n = rB_{n-1} + d, \quad B_n = d\frac{r^n - 1}{r - 1} \ (r \neq 1)$$

■ **Worked example**

Use an illustrative growth rate of 5% per period and deposit 100 units at each period end. B_1=100, B_2=1.05×100+100=205 and B_3=1.05×205+100=315.25. The first deposit earns interest for n−1 periods, while the final one earns none at the time B_n is measured. Thus B_n=100(1.05^n−1)/0.05. For a target of at least 500, B_4=431.0125<500 and B_5=552.563125≥500, so 5 periods first suffice. The balance exceeds the five deposits totalling 500 by 52.563125 units. If the same deposits are made at each period start and the balance is measured at the period end, C_n=1.05(B_n); hence C_3=331.0125, not 315.25.

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Where it goes wrong

- Do not give the final end-of-period deposit an extra interest period.
- A balance includes both deposits and growth; it is not the interest alone.
- An exact target and an at-least target need different integer checks.
- A quoted annual rate cannot simply be used as a per-month factor without an appropriate conversion.

■ Watch out

Not true: Beginning- and end-of-period deposits give the same final balance. A deposit at the start of a period earns one more period of interest, so the final balance is larger.

Applying it

- The illustration assumes a fixed per-period rate, equal deposits, no fees or withdrawals and the stated compounding/payment times.
- If these change, refine the recurrence: B_n=(1+i_n)B_(n−1)+d_n−w_n for end-of-period deposits/withdrawals.
- Keep a separate ledger of payments to identify growth.
- Compare adjacent period balances before claiming the first target crossing; here all balances are nonnegative and B_n−B_(n−1)=0.05B_(n−1)+100>0.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Deposit**: An amount added to an account at a stated time.
- Define the balance immediately after a stated event and give the initial balance.
- Watch for: do not give the final end-of-period deposit an extra interest period.

1 · Matrix transformations, inverses and eigenvalues

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

The method

- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Transform column vectors by left multiplication.
- The rightmost matrix acts first in a product.

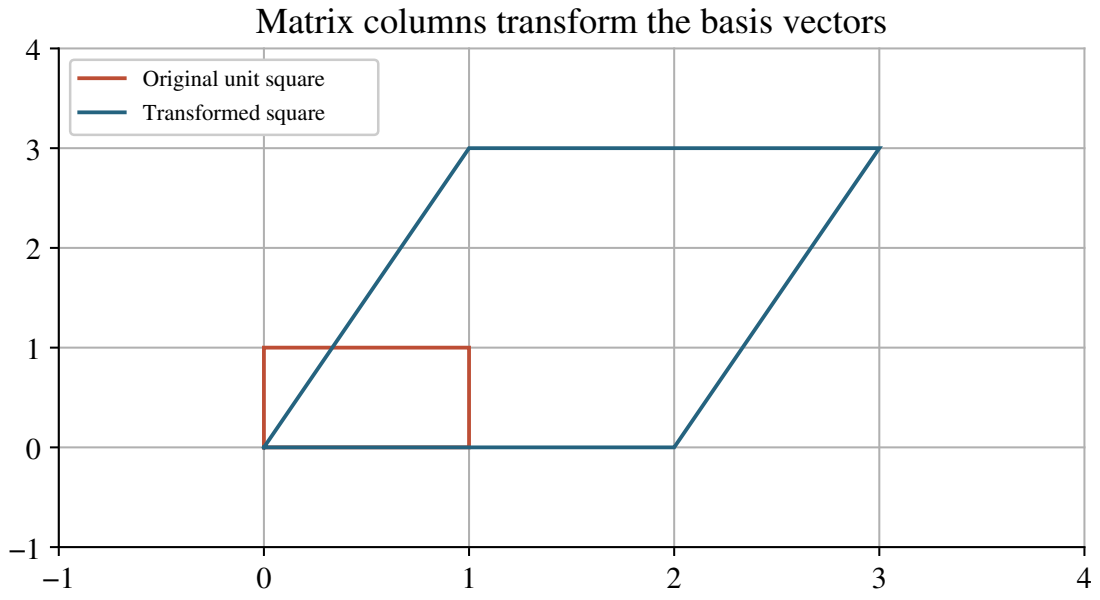
A worked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

■ Worked example

For A=[[2,1],[0,3]], det A=6. A(1,2)=(4,6). Its inverse is [[1/2,-1/6],[0,1/3]]. The characteristic equation is (2-λ)(3-λ)=0, so the eigenvalues are 2 and 3.

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Where it goes wrong

- Matrix multiplication is usually not commutative.
- A determinant of zero means the transformation loses a dimension and has no inverse.
- Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

■ Watch out

Not true: Every square matrix has an inverse. A square matrix with determinant 0 is singular and has no inverse.

Applying it

- Find an eigenvector by solving $(A-\lambda I)v=0$ with $v\neq 0$.
- Explain the geometrical meaning: this vector keeps its line direction under the transformation.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Determinant**: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Watch for: matrix multiplication is usually not commutative.

1 · Complex numbers and roots

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.

The method

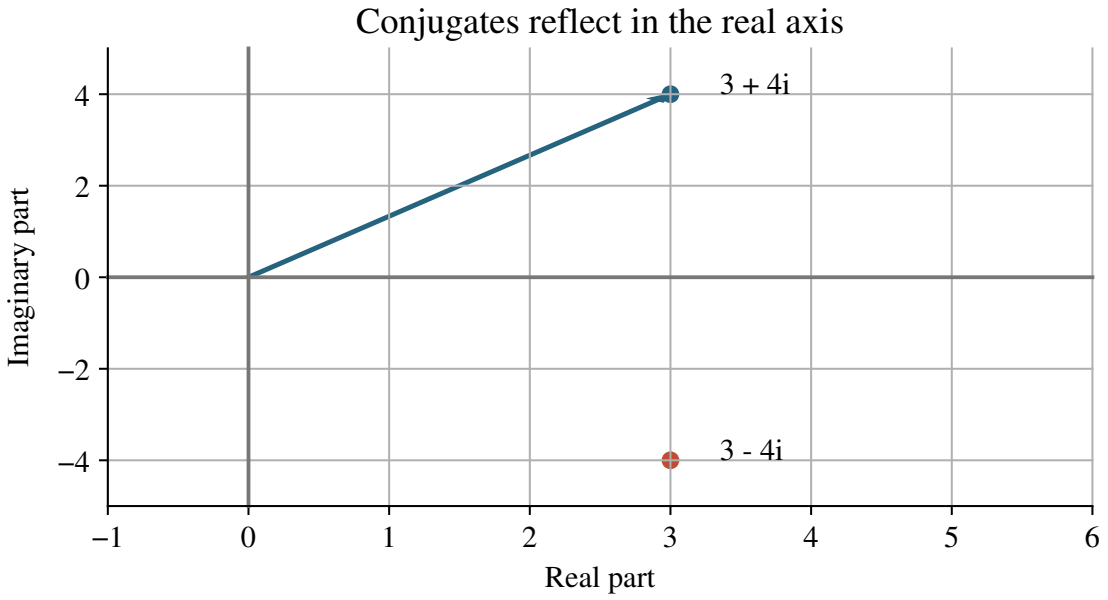
- Write z=a+bi with $i^2=-1$.
- The conjugate is a-bi and z times its conjugate=a^2+b^2.
- The modulus is $\sqrt{(a^2+b^2)}$; the argument needs the correct quadrant.
- Conjugate roots occur for polynomials with real coefficients.

A worked case

$$z = a + bi, \qquad |z| = \sqrt{a^2 + b^2}, \qquad z\overline{z} = a^2 + b^2$$

Worked example

For z=3+4i, |z|=5 and z conjugate=3-4i. The reciprocal is (3-4i)/25. For $x^2-6x+13=0$, the roots are 3±2i. Multiplying by the conjugate makes a complex denominator real.



Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

> ■ **Watch out**
>
> **Not true:** For a+bi, its modulus is a+b. The modulus is $\sqrt{(a^2+b^2)}$: for 3+4i it is 5, not 7.

Applying it

- Draw the real axis horizontally and imaginary axis vertically.
- For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.

In your course

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■ **Key idea**

- **Complex conjugate**: The number obtained by reversing the sign of the imaginary part.
- Write z=a+bi with i^2=-1.
- Watch for: the square root of a sum is not the sum of the square roots.

2 · Domains, inverses and composition

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** 定义域: The set of allowed inputs to a function.

The method

- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f; the intermediate output must be an allowed input to f.

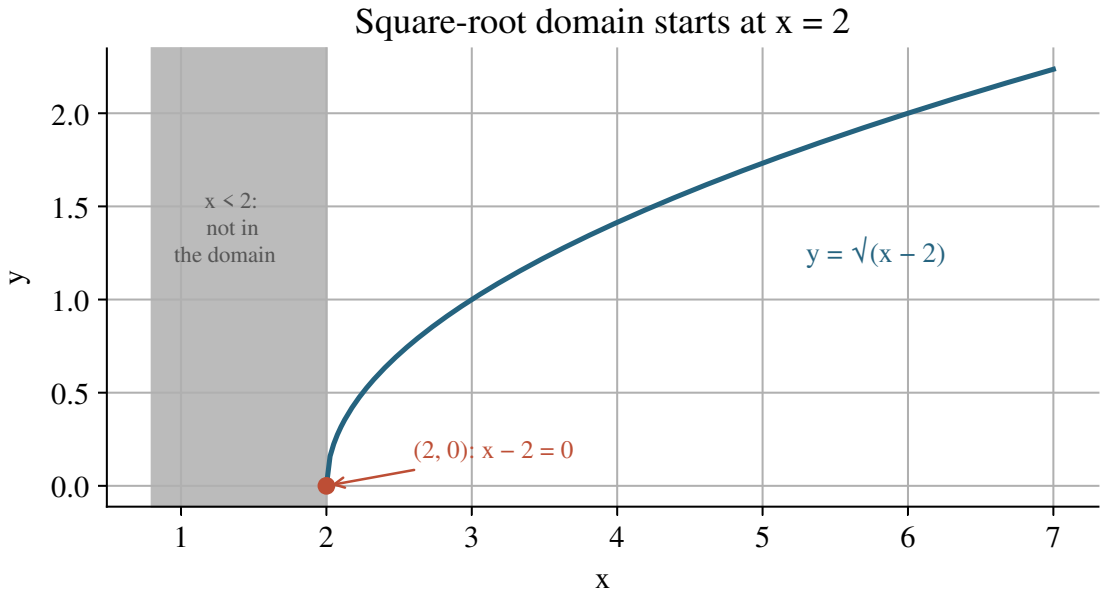
A worked case

$$f(g(x)) = (f \circ g)(x), \qquad f^{-1}(f(x)) = x$$

■ **Worked example**

For f(x)=√(x-2), x≥2 and the range is y≥0. From y=√(x-2), x=y^2+2. Thus f inverse(x)=x^2+2 with x≥0. For g(x)=x+3, fg(1)=f(4)=√2.

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Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

> ■ **Watch out**
>
> **Not true:** Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one (f(1)=f(−1) for x^2), so its domain must be restricted first.

Applying it

- Check f(f inverse(x))=x on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Domain**: The set of allowed inputs to a function.
- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

2 · Regression, financial models and residuals

Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
- This lesson studies **residual** 残差: The observed value minus the value predicted by a fitted model.

The method

- A linear model y=a+bx has intercept a and slope b with contextual units.
- Inspect residuals and the data range.
- For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.

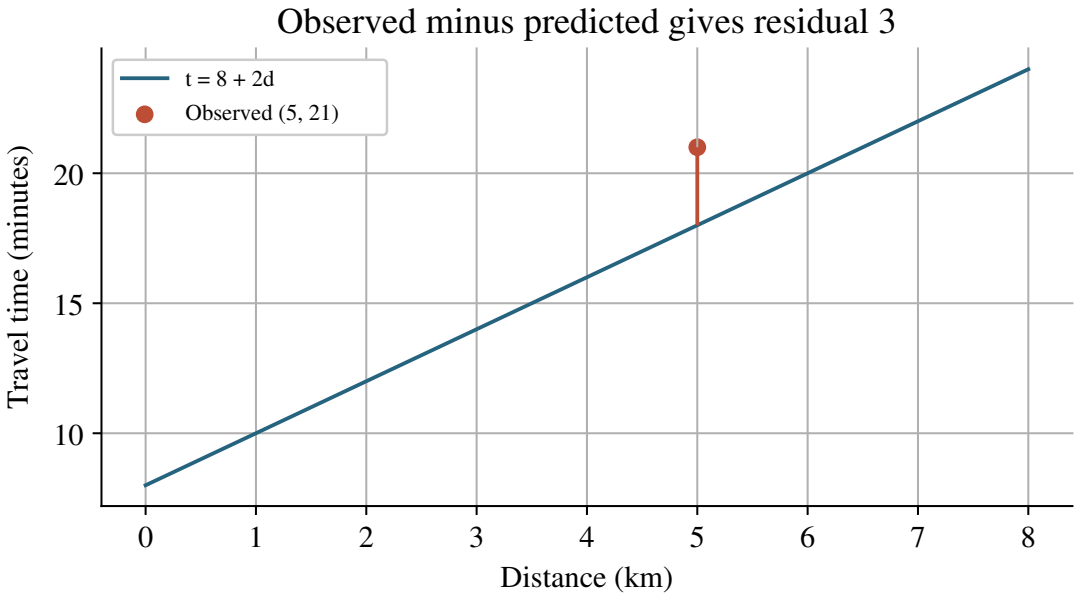
A worked case

$$\widehat{t} = 8 + 2d, \qquad e = t - \widehat{t}$$

■ **Worked example**

The fitted model t=8+2d predicts 18 minutes at d=5 km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes 1000×1.05^3=1157.625 after 3 years.

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Where it goes wrong

- A good fit does not prove a causal mechanism.
- A correlation coefficient measures linear association, not the gradient.
- Calculator output must be translated into a model, checked and interpreted.

■ Watch out

Not true: A high correlation permits reliable extrapolation to any distance. A model fitted to data is reliable only near that data; extrapolating far outside it is unreliable.

Applying it

- In IB AI, record the data source, domain, assumptions and calculator method.
- Compare an alternative model and judge predictions against residuals.
- A financial answer must state payment timing and whether interest is compounded.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.

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■ **Key idea**

- **Residual**: The observed value minus the value predicted by a fitted model.
- A linear model y=a+bx has intercept a and slope b with contextual units.
- Watch for: A good fit does not prove a causal mechanism.

2 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

The method

- For y=Ae^(kt), k is a proportional rate.
- Taking logs gives ln y=ln A+kt.
- Logarithms require positive arguments, and log(x+y) is not log x+log y.

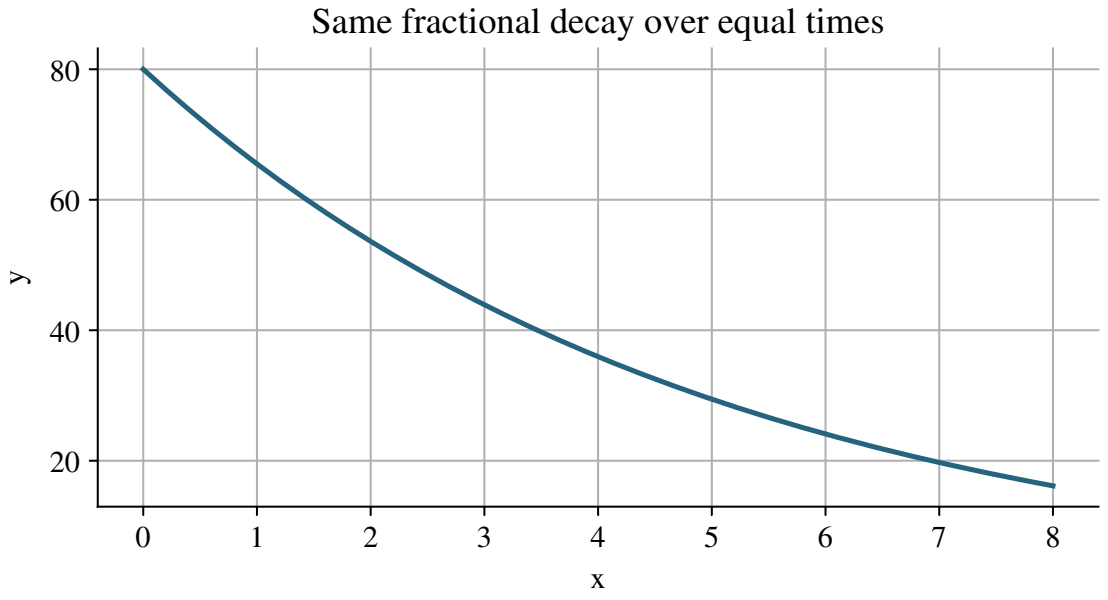
A worked case

$$y = Ae^{kt}, \qquad \ln y = \ln A + kt$$

■ **Worked example**

If y=80e^(-0.2t), y=40 gives e^(-0.2t)=1/2. Hence t=ln2/0.2≈3.466. For 3^x=20, x=ln20/ln3. Plotting ln y against t linearises this exponential model.

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Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

Watch out

Not true: For positive x and y, ln(x+y)=ln x+ln y. The log of a sum has no simple law; the product rule is ln(xy)=ln x+ln y.

Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Half-life**: The time for a decaying quantity to fall to half its initial value.
- For y=Ae^(kt), k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

3 · Angle reasoning, similarity and mensuration

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** 相似比: The multiplier that relates corresponding lengths in similar shapes.

The method

- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

A worked case

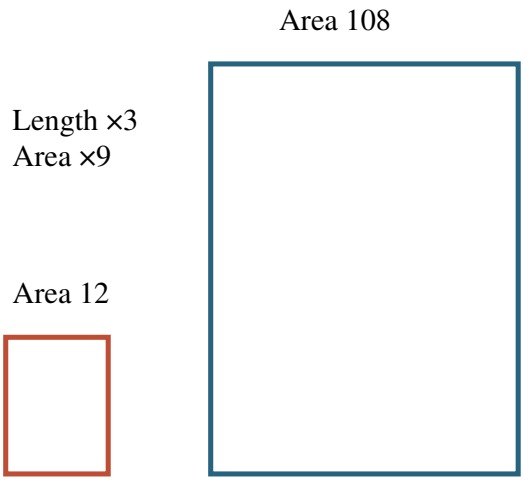
$$\frac{A_2}{A_1} = k^2, \qquad \frac{V_2}{V_1} = k^3$$

■ Worked example

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12\times3^2=108$ cm^2 and a model volume of 8 cm^3 gives $8\times3^3=216$ cm^3. A cylinder with r=3,h=5 has volume $\pi r^2 h=45\pi$.

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A length scale factor is squared for area



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

> ■ **Watch out**
>
> **Not true:** Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

In your course

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■ **Key idea**

- **Scale factor**: The multiplier that relates corresponding lengths in similar shapes.
- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- Watch for: equal angles alone establish similarity, not equal size.

3 · Right triangles and non-right triangles

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.

The method

- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- For other triangles, use the sine or cosine rule, or area=ab sin C/2.

A worked case

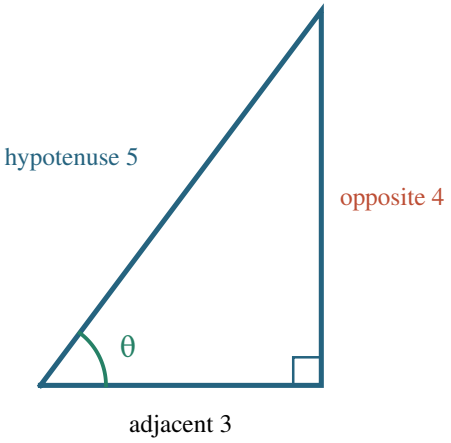
$$a^2 + b^2 = c^2, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ **Worked example**

The ladder length is $c=\sqrt{(3^2+4^2)}=5$ m. Its angle to the ground satisfies $\tan\theta=4/3$, so $\theta\approx53.1°$. With two sides 6 and 8 enclosing 60°, $c^2=6^2+8^2-2\times6\times8\cos60°=52$.

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Label the known sides before choosing a rule



opposite: across from θ
adjacent: next to θ
hypotenuse: across from the right angle

tan θ = 4/3 sin θ = 4/5
cos θ = 3/5

Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

■ Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Hypotenuse**: The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- Watch for: label sides relative to the chosen angle.

3 · Vectors and transformation geometry

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.

The method

- Add corresponding vector components.
- A displacement from A to B is OB-OA.
- Parallel vectors are scalar multiples.
- A vector has magnitude and direction; a distance is a scalar.

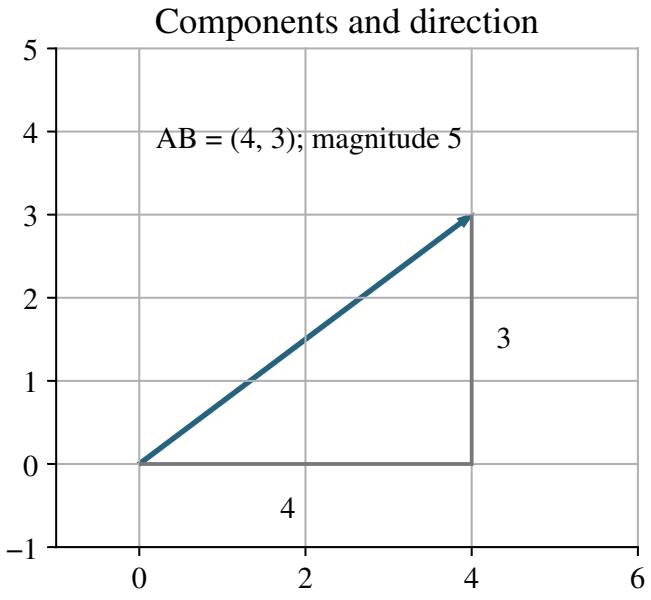
A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \qquad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

■ **Worked example**

With a=(4,1) and b=(1,3), a+b=(5,4). Their scalar product is a · b=4×1+1×3=7. The magnitude of a is √17. For A=(1,2), B=(5,5), AB=(4,3) and |AB|=5.

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Where it goes wrong

- The order of subtraction matters: BA=-AB.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

Watch out

Not true: AB and BA have the same components. BA goes the opposite way to AB, so BA=−AB: every component changes sign.

Applying it

- For a translation, every point uses the same displacement.
- For advanced line problems, distinguish the position vector from a direction vector and solve parameter equations consistently.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Resultant**: The vector sum representing the combined displacement or force.
- Add corresponding vector components.
- Watch for: the order of subtraction matters: BA=-AB.

4 · Data summaries, histograms and interpretation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** 频率密度: Frequency divided by class width, used as histogram height.

The method

- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

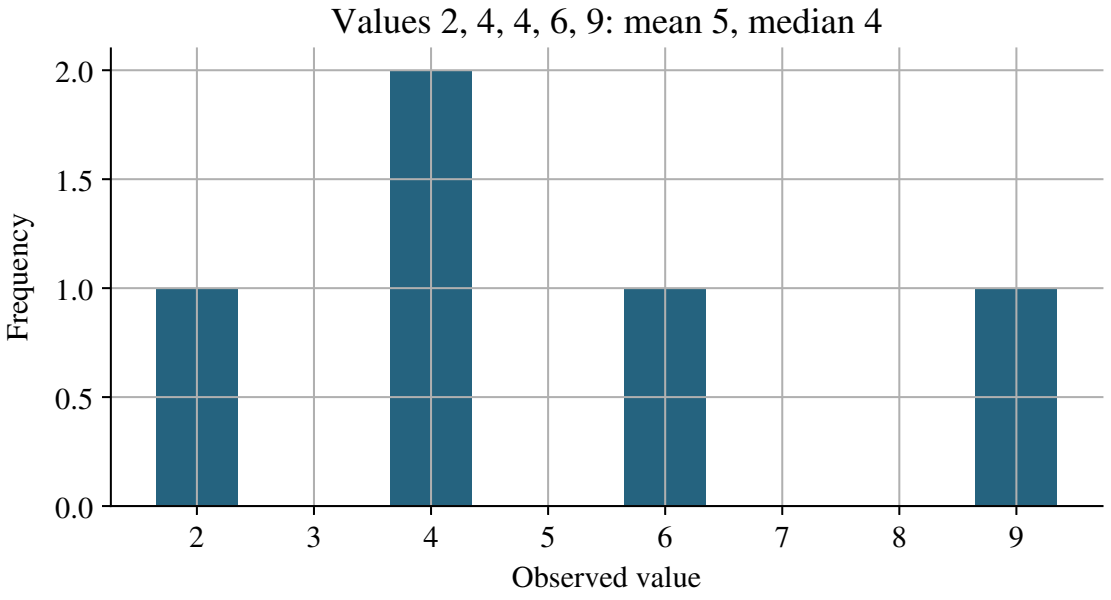
A worked case

$$\overline{x} = \frac{\sum x_i}{n}, \qquad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

■ **Worked example**

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.

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Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

■ Watch out

Not true: A histogram bar's height equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Frequency density**: Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

4 · Probability, trees and conditional reasoning

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.

The method

- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

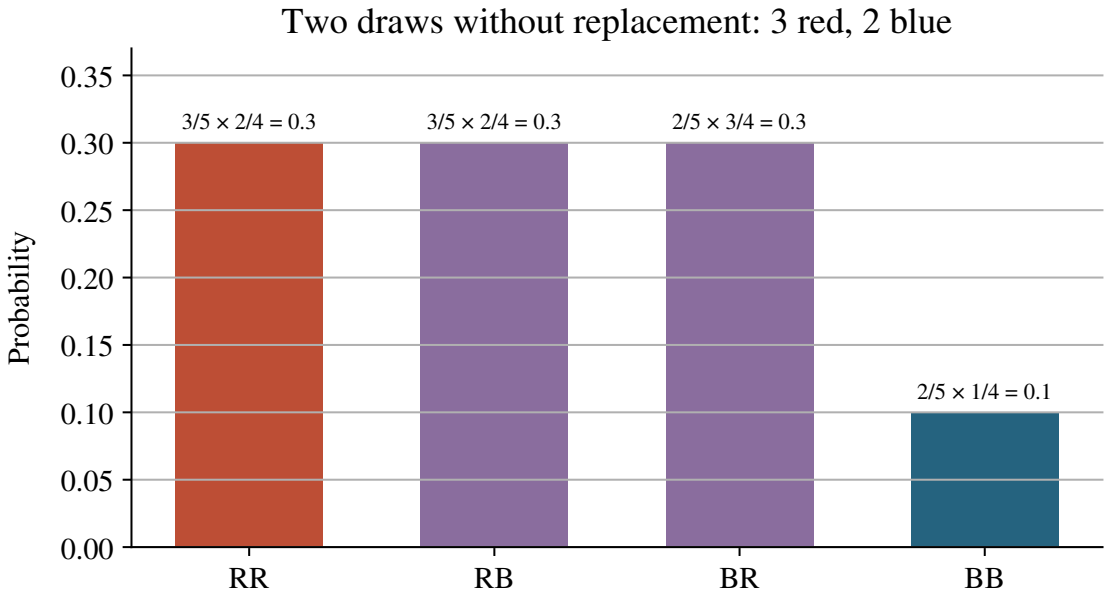
A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

■ Worked example

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/5. If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.

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Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

> ■ **Watch out**
>
> **Not true:** Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B)=0 \neq P(A)P(B)$.

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

4 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.

The method

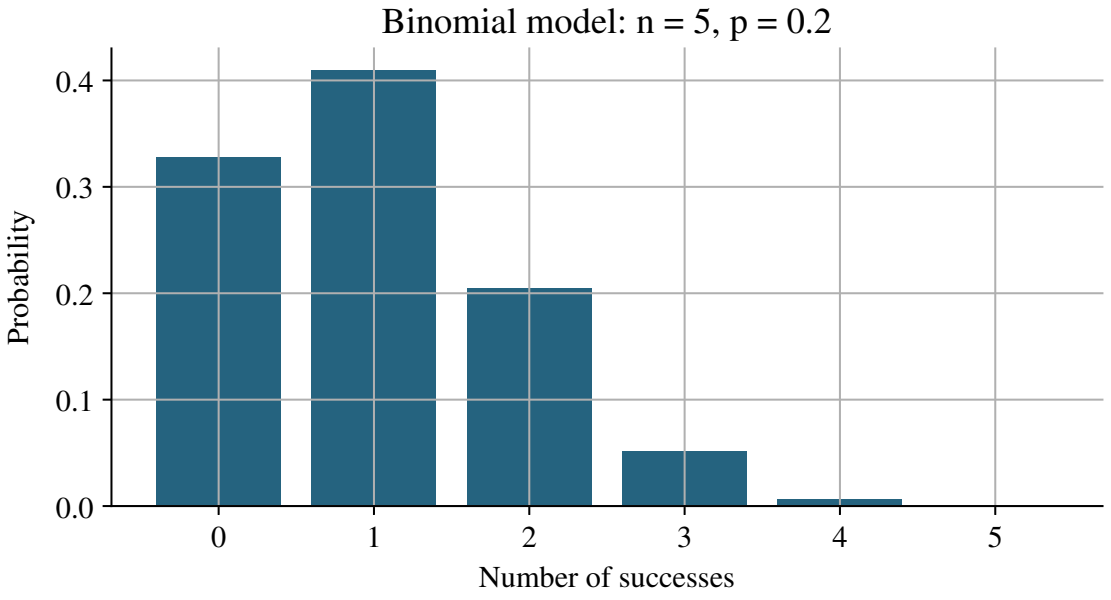
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- E(X)=np and Var(X)=np(1-p).
- For a normal model use z=(x-μ)/σ.
- A Poisson model describes counts with a constant rate and appropriate independence assumptions.

A worked case

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad X \sim B(n, p)$$

■ **Worked example**

For X binomial(5,0.2), P(X=0)=0.8^5=0.32768, E(X)=1 and Var(X)=0.8. For a normal quantity with μ=100,σ=15, the value 130 has z=2. A Poisson mean of 3 per hour gives mean 6 over two hours.



Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

■ Watch out

Not true: Every count of successes has a binomial distribution regardless of dependence. The binomial model needs independent trials with a fixed probability; dependent trials break it.

Applying it

- Write the event as an inequality before using calculator distribution functions.
- Distinguish $P(X<k)$, $P(X\leq k)$ and a tail complement.
- State assumptions in context.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.

■ Key idea

- **Expected value**: The probability-weighted mean of a random variable.
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- Watch for: not every count is binomial: changing p or dependence can invalidate it.

4 · Hypothesis testing and contextual conclusions

Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
- This lesson studies **significance level** 显著性水平: The chosen probability threshold for rejecting a null hypothesis.

The method

- State H_0 and H_1 in population parameters before inspecting the outcome.
- Calculate the appropriate tail probability under H_0.
- Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.

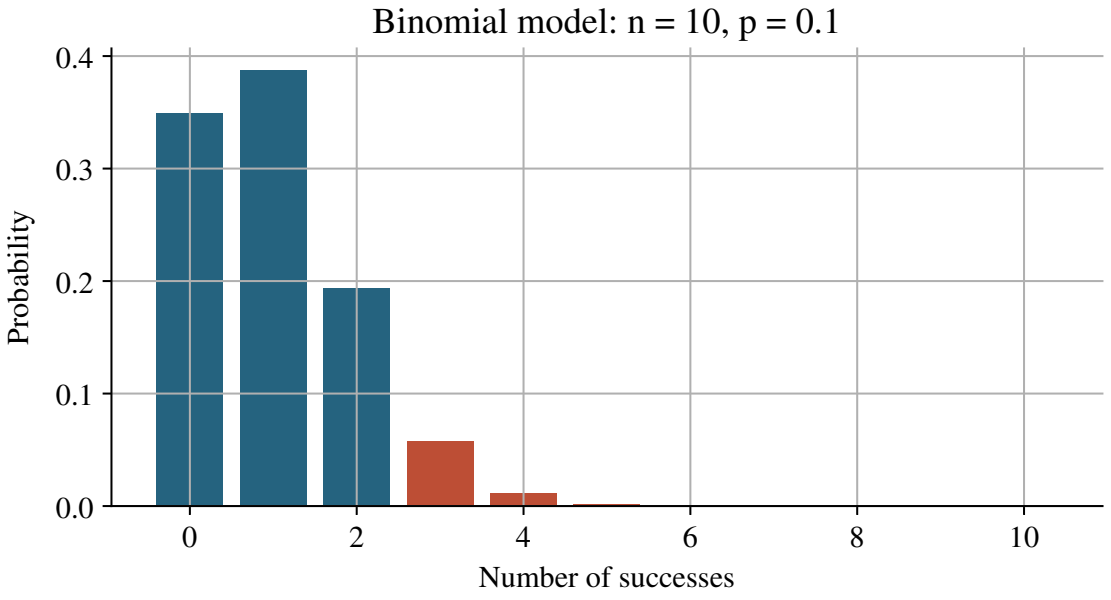
A worked case

$$H_0 : p = p_0, \qquad H_1 : p > p_0$$

■ Worked example

For H_0:p=0.1 and H_1:p>0.1 with n=10, observing at least 3 defects has probability 1-(0.9^10+10×0.1×0.9^9+45×0.1^2×0.9^8)≈0.070191. At 5%, this is insufficient evidence that the defect rate exceeds 10%.

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Where it goes wrong

- Failing to reject H_0 is not proof that H_0 is true.
- Choose the tail from H_1, not from whichever tail gives a small result.
- Statistical significance does not measure the practical size of an effect.

> ■ **Watch out**
>
> **Not true:** Failing to reject a null hypothesis proves it is true. Not rejecting the null hypothesis only means there is not enough evidence against it.

Applying it

- Finish with a sentence about the population and the original claim.
- State the model's assumptions and consider whether the sampling procedure supports them.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Significance level**: The chosen probability threshold for rejecting a null hypothesis.
- State H_0 and H_1 in population parameters before inspecting the outcome.
- Watch for: failing to reject H_0 is not proof that H_0 is true.

4 · Confidence intervals, chi-square and regression

How uncertain is the sample mean?

- Two samples give different means. We need a statement about sampling uncertainty, not a promise that one sample equals the population.
- This lesson studies **confidence interval** 置信区间: An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate.

The method

- For a normal mean with known σ, a 95% interval is sample mean$\pm 1.96\sigma/\sqrt{}$n.
- For chi-square, sum (observed-expected)^2/expected with the correct degrees of freedom.
- Choose a test that fits the data and assumptions.

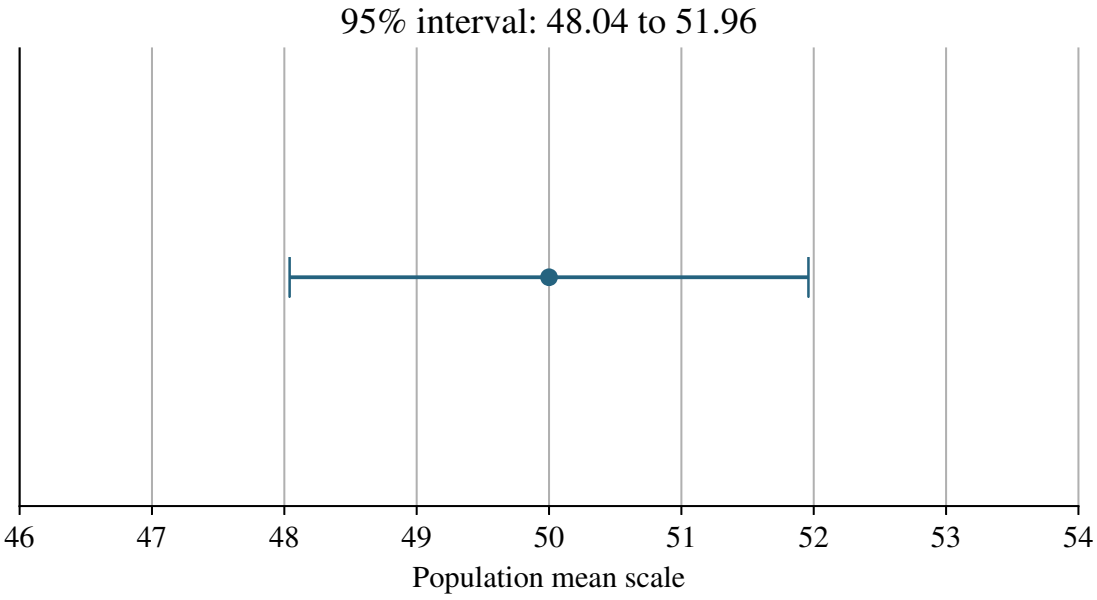
A worked case

$$\overline{x} \pm 1.96\frac{\sigma}{\sqrt{n}}, \qquad \chi^2 = \sum \frac{(O-E)^2}{E}$$

■ **Worked example**

With mean=50,σ=10,n=100, standard error=1 and the 95% interval is [48.04,51.96]. For observed counts 30 and 20 against expected 25 and 25, chi-square=25/25+25/25=2, before deciding degrees of freedom and the rejection threshold.

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Where it goes wrong

- A 95% confidence procedure does not give a 95% probability that a fixed population mean moves within this one interval.
- Expected counts and estimated parameters affect the validity and degrees of freedom of chi-square tests.

■ Watch out

Not true: A strong correlation alone establishes a causal relationship. Correlation shows association; a third variable or chance can explain it, so causation needs an experiment or other evidence.

Applying it

- Report uncertainty in the context.
- In regression, inspect residuals, avoid unjustified extrapolation, and distinguish a fitted relationship from a causal explanation.
- S3 and IB AI HL require different verified scope boundaries.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Confidence interval**: An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate.
- For a normal mean with known σ, a 95% interval is sample mean$\pm 1.96\sigma/\sqrt{n}$.
- Watch for: A 95% confidence procedure does not give a 95% probability that a fixed population mean moves within this one interval.

5 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

The method

- For y=ax^n, dy/dx=anx^(n-1).
- A stationary point satisfies dy/dx=0.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

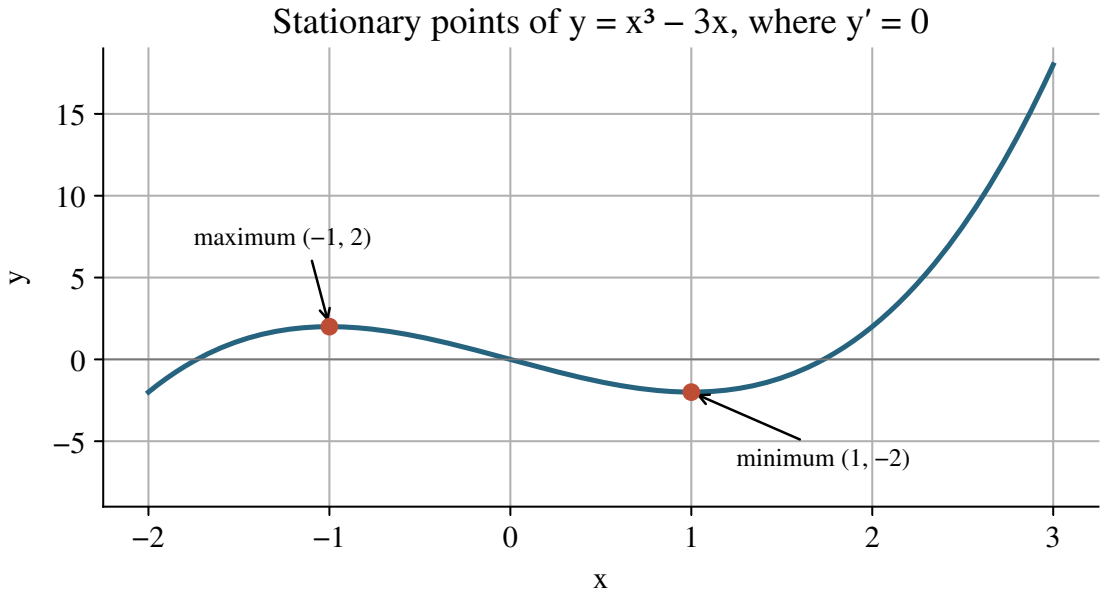
A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \qquad f'(x) = 0$$

Worked example

For $y=x^3-3x$, $dy/dx=3x^2-3$. At x=1, the gradient is 0 and y=-2. The second derivative is 6x, positive at x=1, so this is a local minimum. At x=-1, y=2 and the second derivative is negative, giving a local maximum.

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Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

■ Watch out

Not true: Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

Applying it

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For y=ax^n, dy/dx=anx^(n-1).
- Watch for: A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.

5 · Root finding and numerical integration

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.

The method

- A continuous function with opposite signs at two endpoints has a root between them.
- Newton's method uses x next=x-f(x)/f prime(x), with a nonzero derivative.
- The trapezium rule approximates a definite integral using endpoint heights.

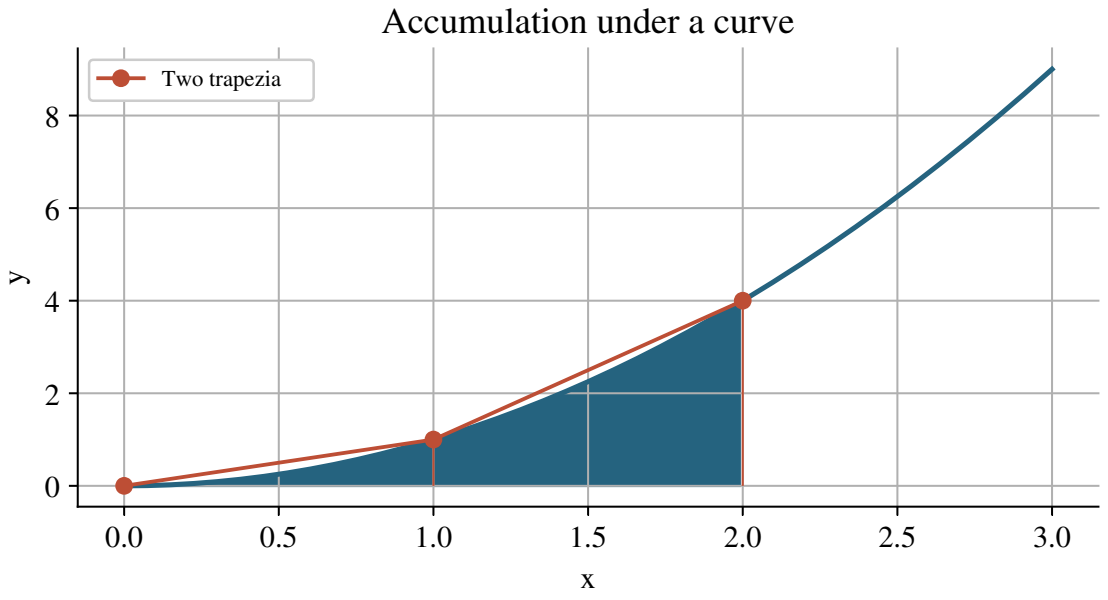
A worked case

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Worked example

For f(x)=x^2-2 and x_0=1.5, Newton gives x_1=1.5-(2.25-2)/3=1.4166667. With $y=x^2$ on [0,2] and two equal strips, h=1 and trapezium area=(1/2)(0+2×1+4)=3; the exact area is 8/3.

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Where it goes wrong

- A sign change across a discontinuity does not prove a root.
- Iteration can diverge or cycle.
- The trapezium rule's overestimate or underestimate depends on curvature, not just whether the function increases.

■ Watch out

Not true: Every sign change proves a root, including one across a discontinuity. A sign change shows a root only if the function is continuous there: $1/x$ changes sign across $x=0$ but has no root.

Applying it

- Give a stopping criterion and report sensible accuracy.
- Confirm the approximated root by a sign bracket around the stated rounded answer; explain any failure of the chosen iteration.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Iteration**: A repeated update in which each new approximation is calculated from the previous one.
- A continuous function with opposite signs at two endpoints has a root between them.
- Watch for: A sign change across a discontinuity does not prove a root.

5 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.

The method

- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- For uv, differentiate to u prime v+uv prime.
- For u/v, use (u prime v-uv prime)/v^2, where v≠0.

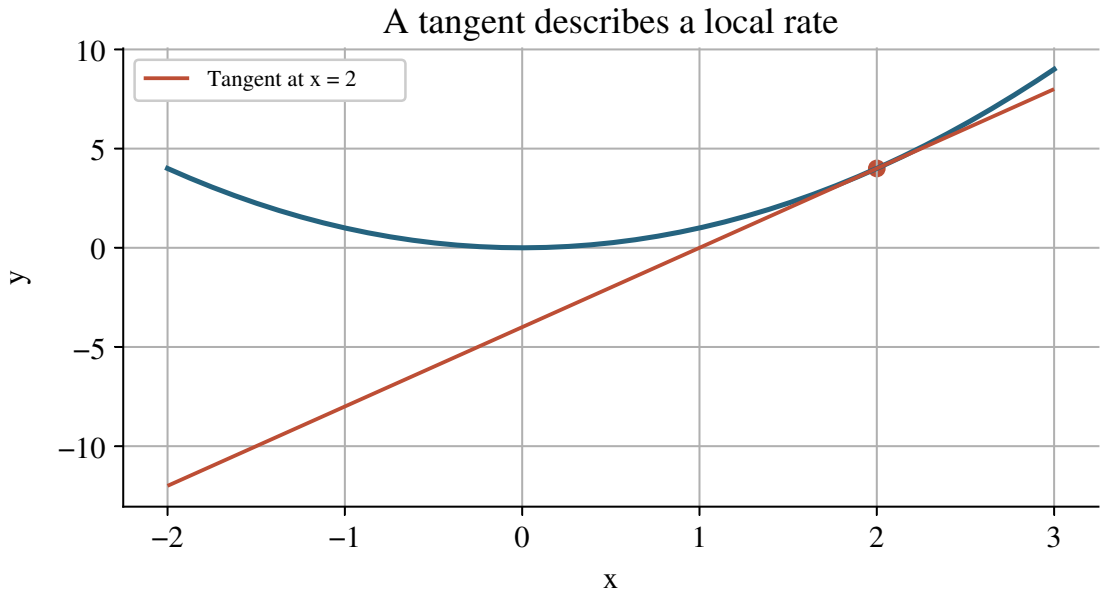
A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

■ Worked example

For y=(3x+1)^4, y prime=4(3x+1)^3×3=12(3x+1)^3. At x=0, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: 2x+2y y prime=0, so y prime=-x/y when y≠0.

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Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

■ Watch out

Not true: The derivative of u(x)v(x) is u prime times v prime. The product rule is $(uv)'=u'v+uv'$: for $x \cdot x$ the derivative is 2x, not $1\times1=1$.

Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Chain rule**: The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- Watch for: A derivative of a product is not the product of the derivatives.

5 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.

The method

- Use substitution when an inner derivative appears as a factor.
- By parts, integral u v prime =uv-integral u prime v.
- For a rational function, divide first if needed, then decompose into partial fractions.

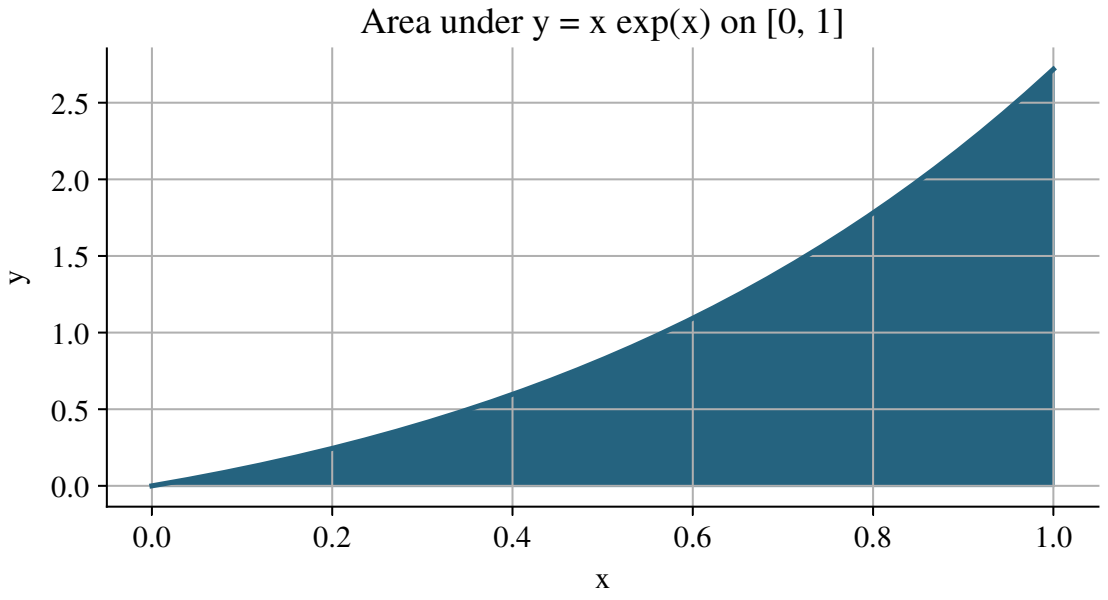
A worked case

$$\int uv'\,dx = uv - \int u'v\,dx$$

■ **Worked example**

For integral xe^x dx, take u=x and v prime=e^x. Then the integral is xe^x-e^x+C. For integral 2x/(x^2+1) dx, substitute w=x^2+1, dw=2x dx; the answer is ln(x^2+1)+C.

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Where it goes wrong

- Choosing u and v prime well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

> ■ **Watch out**
>
> **Not true:** The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: $\int x \cdot x\,dx = x^3/3$, not $(x^2/2)(x^2/2)$.

Applying it

- Check by differentiating.
- For volumes of revolution around the x-axis use $V = \pi \int y^2\,dx$; do not confuse the square of a function with the integral of the function.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Integration by parts**: An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

5 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

The method

- For dy/dx=ky, separate variables: dy/y=k dx, giving y=Ae^(kx).
- Use an initial condition to find A.
- Euler's method takes y next=y+h f(x,y), with a chosen step h.

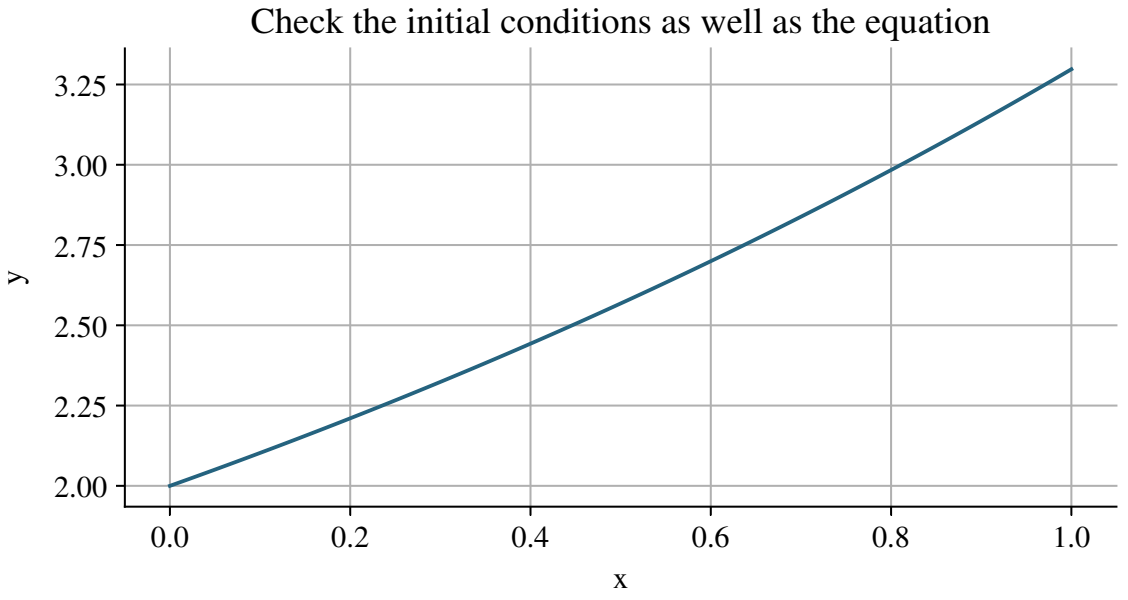
A worked case

$$\frac{dy}{dx} = ky, \qquad y = Ae^{kx}$$

■ Worked example

With dy/dx=0.5y and y(0)=2, y=2e^(0.5x). Euler with h=0.2 gives y(0.2)≈2+0.2×1=2.2 and y(0.4)≈2.2+0.2×1.1=2.42. The exact second value is about 2.4428.

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Where it goes wrong

- An initial condition determines the integration constant; it is not a replacement for integrating.
- Dividing by y can omit an equilibrium solution y=0.
- Euler accuracy depends on step size and the equation.

> ■ **Watch out**
>
> **Not true:** Every step size gives exactly the same Euler approximation. Euler's method depends on the step size; smaller steps usually give a more accurate value.

Applying it

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions.
- This is Further Pure scope; check the named unit.

In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Initial condition**: A specified value that selects a particular solution from a family.
- For dy/dx=ky, separate variables: dy/y=k dx, giving y=Ae^(kx).
- Watch for: an initial condition determines the integration constant; it is not a replacement for integrating.

IA · Planning a mathematical exploration

What makes an investigation mathematical?

- A student wants to predict queue waiting time at the school canteen. A useful exploration needs a question, data and mathematics the student can explain.
- This lesson studies **assumption** 假设: A condition adopted to make a mathematical model possible and whose effect should be evaluated.

The method

- Frame a focused question with measurable variables and a feasible method.
- Keep a dated record of decisions, sources and mathematical reasoning.
- Use technology to support analysis, and explain every important output in your own words.

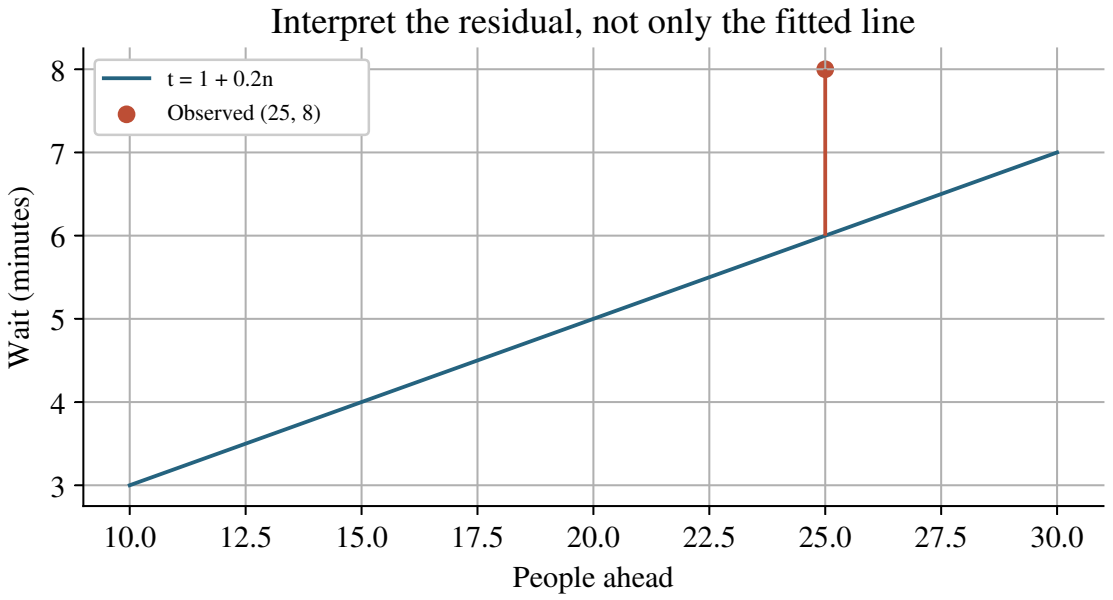
A worked case

$$\widehat{t} = 1 + 0.2n, \qquad e = t - \widehat{t}$$

■ **Worked example**

A pilot records mean waiting times of 3,5,7 minutes when 10,20,30 people are ahead. A first model is t=1+0.2n. At n=25 it predicts 6 minutes. An actual wait of 8 gives residual 2; investigate varying service rates rather than hiding the mismatch.

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Where it goes wrong

- Do not copy a sample exploration, invent data or manufacture personal engagement.
- A prediction is not evidence until it is compared with observations.
- Consent and privacy matter when collecting school data.

■ Watch out

Not true: The school-supervised mathematical exploration can be replaced by a timed written mock exam. The exploration is an extended piece of written work assessed on its own criteria; a timed exam tests something different.

Applying it

- The school supervises topic approval, checkpoints, authenticity and assessment.
- For the current 2021 course use the current school's guide and criteria; the 2029 inquiry criteria belong to a different assessment version.

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In your course

- Applications and Interpretation HL, the course first assessed in 2021.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Assumption**: A condition adopted to make a mathematical model possible and whose effect should be evaluated.
- Frame a focused question with measurable variables and a feasible method.
- Watch for: do not copy a sample exploration, invent data or manufacture personal engagement.

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