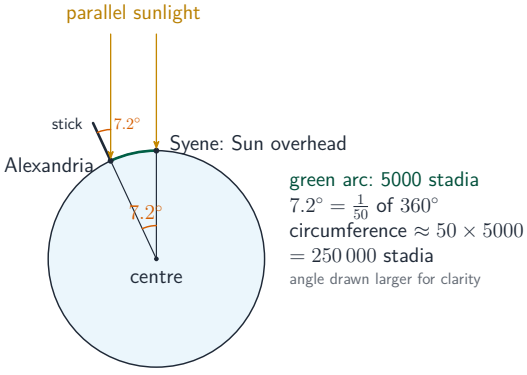
Geometry and trigonometry

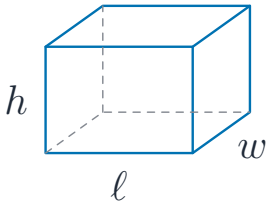
Topic 3 · SL

Measuring the Earth with a shadow

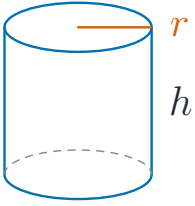


One angle and one distance gave the size of the Earth.

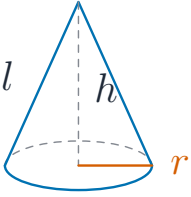
Four solids, their dimensions



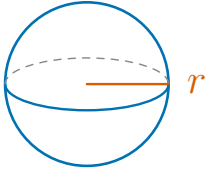
cuboid



cylinder



cone



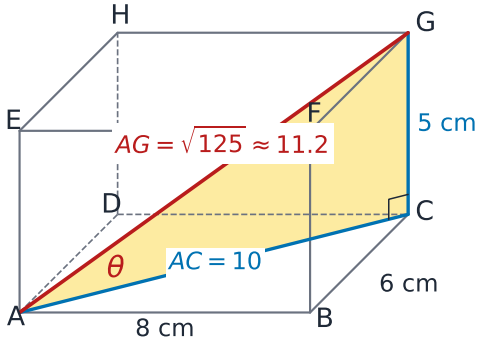
sphere

- Name r, h and slant l before using a formula.
- **Cone**: $V = \frac{1}{3}\pi r^2 h$, curved area πrl.

Solids

Solid	**Volume**
Pyramid	$\frac{1}{3} \times$ base $\times\, h$
Cone	$\frac{1}{3}\pi r^2 h$; curved surface $\pi r l$
Sphere	$\frac{4}{3}\pi r^3$; surface $4\pi r^2$
Cone example	$96\pi \approx 302$ cm^3

A line and a plane



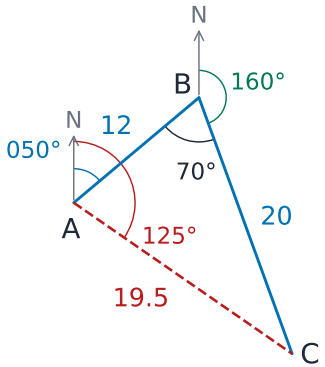
$\tan\theta = \frac{5}{10}$
$\theta \approx 26.6°$

- Find the right-angled triangle first.
- $AC = 10$, $CG = 5$, $\tan\theta = \frac{5}{10}$.

Sine and cosine rules

Rule	Formula
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$
Area	$\frac{1}{2}ab\sin C$

Bearings



1 angle at B

back-bearing B→A = 050° + 180° = 230°

ABC = 230° − 160° = 70°

2 cosine rule for AC

$AC^2 = 12^2 + 20^2 - 2 \cdot 12 \cdot 20 \cdot \cos 70°$

$= 379.8$, so $AC = 19.5$ km

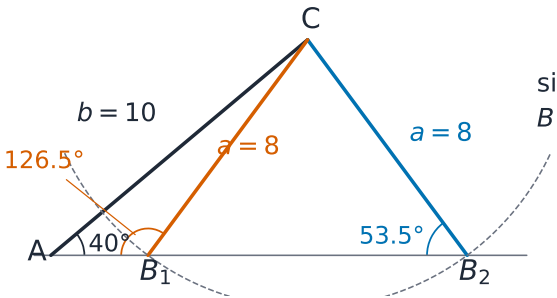
3 sine rule, then the bearing

$\sin A = 20 \sin 70° / 19.5 \rightarrow A = 74.6°$

bearing of C = 050° + 74.6° ≈ 125°

- 12 km on 050°, then 20 km on 160°: find AC and the **bearing** of C from A.

The ambiguous case

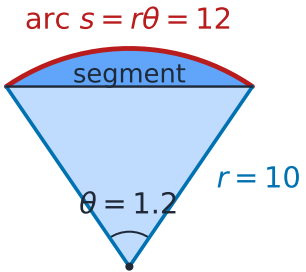


$\sin B = \frac{10 \sin 40°}{8}$

$B = 53.5°$ or $126.5°$

- $\sin B = \sin(180° - B)$.
- Check the obtuse angle leaves room.

Arcs and sectors



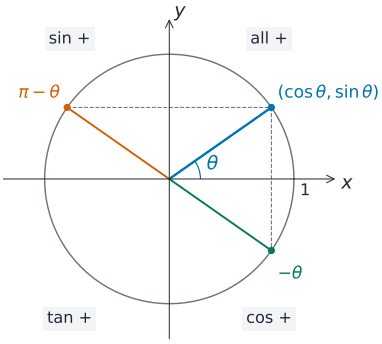
$$\text{sector} = \tfrac{1}{2}r^2\theta = 60$$

$$\text{triangle} = \tfrac{1}{2}r^2\sin\theta \approx 46.6$$

$$\text{segment} = \tfrac{1}{2}r^2(\theta - \sin\theta) \approx 13.4$$

- π rad $= 180°$.
- $s = r\theta$, $A = \frac{1}{2}r^2\theta$, θ in radians.

The unit circle



- $\sin(\pi - \theta) = \sin\theta$.
- $\cos(-\theta) = \cos\theta$.

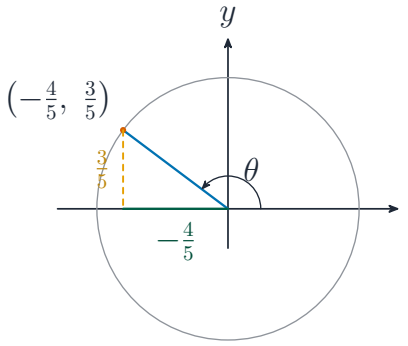
Identities

1. **Cosine.** $\cos\theta = -\frac{4}{5}$
2. **Double angle.** $\sin 2\theta = -\frac{24}{25}$
3. **Cosine form.** $\cos 2\theta = 1 - 2\sin^2\theta = \frac{7}{25}$

Decision

$\sin^2\theta + \cos^2\theta = 1$

An obtuse angle on the unit circle



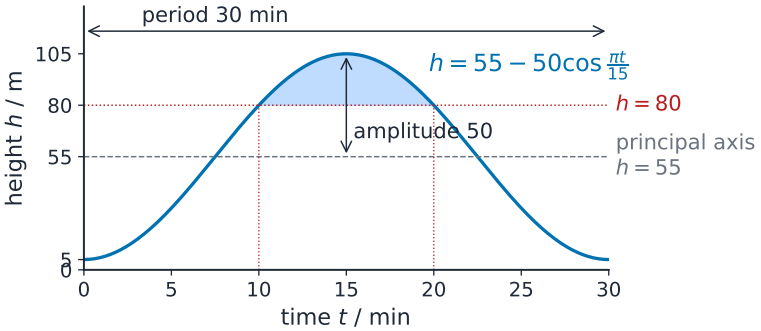
second quadrant: $\cos\theta < 0$

$\cos\theta = -\frac{4}{5},\ \tan\theta = -\frac{3}{4}$

$\sin 2\theta = -\frac{24}{25},\ \cos 2\theta = \frac{7}{25}$

- The point $(-\frac{4}{5}, \frac{3}{5})$: $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$.
- $\sin 2\theta = -\frac{24}{25}$, $\cos 2\theta = \frac{7}{25}$.

Modelling a Ferris wheel



- **Amplitude** $|a|$, **period** $\frac{2\pi}{b}$, axis $y = d$.
- Above 80 m for $10 < t < 20$.

Every solution

1. **Interval.** $0 \leq 2x \leq 4\pi$
2. **Solve.** $2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$
3. **Divide.** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Decision

Four answers, not two

Quadratic in sine

1. **Identity.** $2\sin^2 x - 3\sin x + 1 = 0$
2. **Factorise.** $(2\sin x - 1)(\sin x - 1) = 0$
3. **Solve.** $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

Decision

One function, then factorise

Check: how many solutions?

Your turn

How many solutions has $\tan 3x = 1$ for $0 \leq x \leq \pi$?

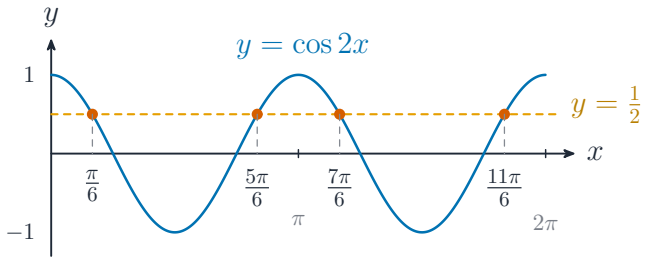
A. 3

B. 1

C. 2

D. 6

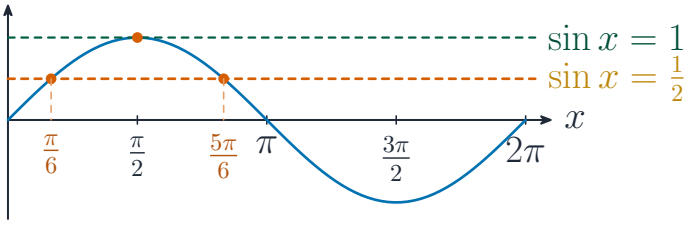
Double the angle, double the solutions



two full turns of $u = 2x$ in $[0, 2\pi]$: four solutions, not two

- $\cos 2x = \frac{1}{2}$ has four solutions in $[0, 2\pi]$.
- $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

Solving $\sin x = \frac{1}{2}$



$$(2\sin x - 1)(\sin x - 1) = 0$$

Read every solution in the interval

Exam tips

After the **sine rule**, test the obtuse angle too.

- Radians for arcs and calculus.
- Stretch the interval for $\sin kx$.
- Keep exact values on Paper 1.

Check yourself

Try these

In triangle ABC, $A = 30°$, $a = 6$, $b = 10$. Find both possible values of B.

A sector has radius 8 cm and area 40 cm^2. Find its angle and perimeter.

Solve $2\sin^2 x = 3\cos x$ for $0 \le x \le 2\pi$.