

Exercise walk-through

5.1

Derivatives and stationary points

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

5.1 Derivatives and stationary points

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS derivative

Objective

Derivatives and stationary points. Supported SL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.

■ Choose the mathematical structure

- For $y = ax^n$, $dy/dx = anx^{n-1}$. A stationary point satisfies $dy/dx = 0$. Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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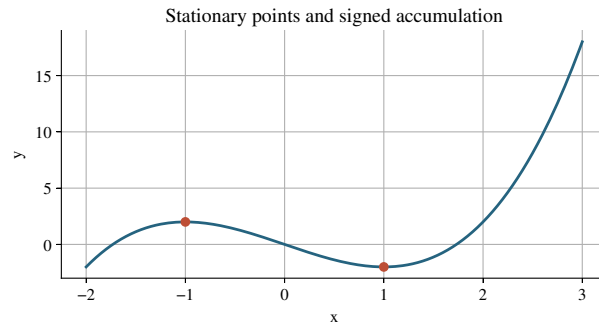
■ Work through a checked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $y=x^3-3x$, $dy/dx=3x^2-3$. At $x=1$, the gradient is 0 and $y=-2$. The second derivative is $6x$, positive at $x=1$, so this is a local minimum. At $x=-1$, $y=2$ and the second derivative is negative, giving a local maximum.



■ **Test a tempting shortcut**

- A zero derivative does not always mean a maximum or minimum; $y=x^3$ is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there. At advanced level, connect the derivative to rates and optimization with a valid domain.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches SL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1·2.3

2.1 For $y=x^3-3x$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

2.2 Find the y-coordinate of the stationary point at $x=1$. Show the relation, substitution and units. [2]

2.3 For $y=4x^3$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For $y=x^3-3x$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

2.1 $dy/dx=3x^2-3$. At $x=2$ it is $12-3=9$.

2.2 Find the y-coordinate of the stationary point at $x=1$. Show the relation, substitution and units. [2]

2.2 Substitute into the original curve: $y(1)=1-3=-2$.

2.3 For $y=4x^3$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

2.3 The derivative is $12x^2$; at 2 it is $12 \times 4 = 48$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Every point with zero derivative is a local maximum or minimum. [2]

2.5 Which description correctly defines derivative? Choose one answer.

- **A** The instantaneous rate of change, also the gradient of a tangent.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail [1]
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3.1 For $y=x^3-3x$ on $[-2, 2]$, find the absolute maximum and minimum, including all candidates. [4]

Exercises 2.4-3.1

2.4 Decide whether this claim is true and justify: Every point with zero derivative is a local maximum or minimum. [2]

2.4 False. A zero derivative does not always mean a maximum or minimum: $y=x^3$ is

2.5 Which description correctly defines derivative? Choose one answer.

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2.5 A The instantaneous rate of change, also the gradient of a tangent.

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3.1 Derivative $3x^2-3=0$ at ± 1 . Values $y(-2)=-2$, $y(-1)=2$, $y(1)=-2$, $y(2)=2$. Maximum 2 at $-1, 2$; minimum -2 at $-2, 1$. Checking only stationary points misses endpoint ties.