

Exercise walk-through

4.2

Probability, trees and conditional reasoning

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

4.2 Probability, trees and conditional reasoning

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS conditional probability

Objective

Probability, trees and conditional reasoning. Supported SL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

■ **Choose the mathematical structure**

- Multiply along a tree branch and add disjoint branches. With replacement, the composition stays fixed. Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

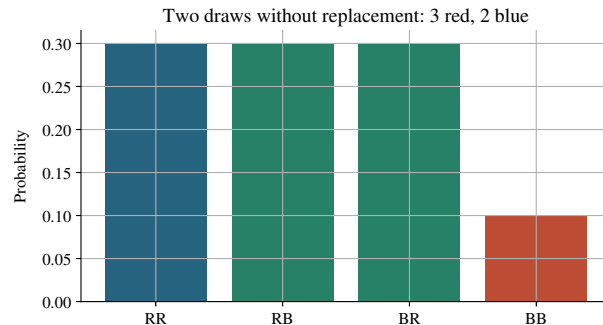
■ **Work through a checked case**

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

Without replacement, $P(\text{two red}) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$. $P(\text{one of each}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$.
If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



■ **Test a tempting shortcut**

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Mutually exclusive events with positive probabilities must be independent. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- A two-way table makes the restricted denominator visible. Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

- Current first-assessment-2021 Analysis and Approaches SL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The probability of an event after restricting the sample space to a stated condition. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1-2.3

2.1 Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

2.2 Find $P(A \text{ given } B)$ if $P(A \cap B) = 0.12$ and $P(B) = 0.3$. Show the relation, substitution and units. [2]

2.3 Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

2.1 Without replacement, $P(RR)=3/5 \times 2/4=0.3$.

2.2 Find $P(A \text{ given } B)$ if $P(A \cap B)=0.12$ and $P(B)=0.3$. Show the relation, substitution and units. [2]

2.2 $P(A|B)=P(A \cap B)/P(B)=0.12/0.3=0.4$.

2.3 Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

2.3 Add the two orders: $3/5 \times 2/4 + 2/5 \times 3/4 = 0.6$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Mutually exclusive events with positive probabilities must be independent. [2]

2.5 Which description correctly defines conditional probability? Choose one answer.

- **A** The probability of an event after restricting the sample space to a stated condition.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Mutually exclusive events with positive probabilities must be independent. [2]

2.4 False. Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.

2.5 Which description correctly defines conditional probability? Choose one answer.

- **A** The probability of an event after restricting the sample space to a stated condition.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

2.5 A The probability of an event after restricting the sample space to a stated condition.

Exercise 3.1

3.1 A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

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3.1 A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

3.1 $P(RR) = \frac{3}{5} \times \frac{2}{4} = 0.3$. $P(\text{at least one } R) = 1 - \frac{2}{5} \times \frac{1}{4} = 0.9$. Conditional probability $= 0.3 / 0.9 = 1/3$. The conditioning changes the sample space.