

Calculus

International Baccalaureate · IB Diploma · Mathematics: Analysis
and Approaches · SL

Scope and prerequisites

This handout covers selected SL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Derivatives and stationary points

■ What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.

■ The method

- For $y = ax^n$, $dy/dx = anx^{n-1}$.
- A stationary point satisfies $dy/dx = 0$.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

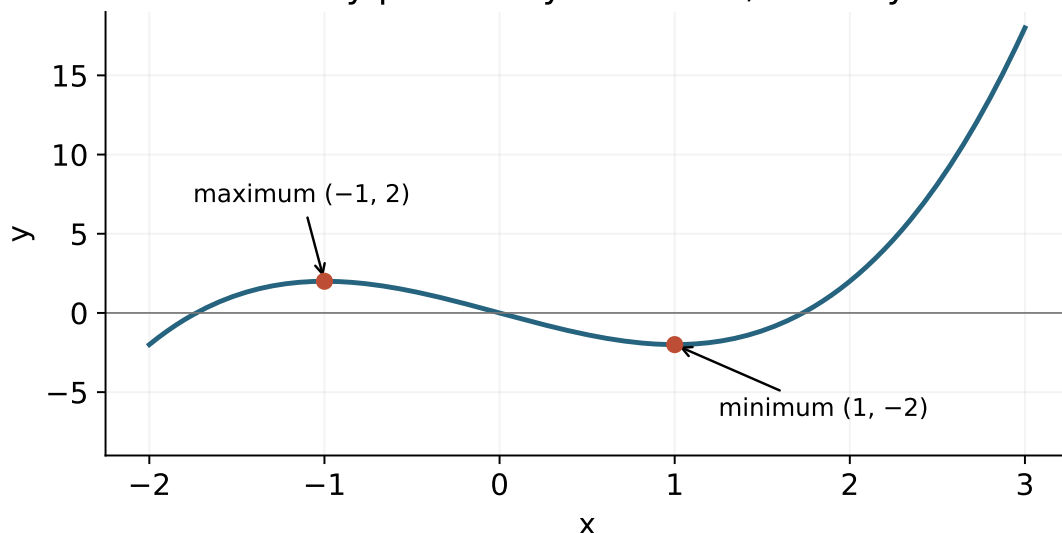
■ A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

Example:

For $y = x^3 - 3x$, $dy/dx = 3x^2 - 3$. At $x = 1$, the gradient is 0 and $y = -2$. The second derivative is $6x$, positive at $x = 1$, so this is a local minimum. At $x = -1$, $y = 2$ and the second derivative is negative, giving a local maximum.

Stationary points of $y = x^3 - 3x$, where $y' = 0$



$$y' = 3x^2 - 3 = 0 \text{ at } x = \pm 1: \text{ maximum } (-1, 2), \text{ minimum } (1, -2)$$

■ Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

Not true: Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

■ Applying it

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Derivative:** The instantaneous rate of change, also the gradient of a tangent.
- For $y = ax^n$, $dy/dx = anx^{n-1}$.
- Watch for: A zero derivative does not always mean a maximum or minimum: $y = x^3$ is stationary at 0 but continues increasing.

■ Write from a derivative of 6

- For $y = x^2 - 4x$, find the derivative at $x = 5$ and the y-coordinate of the stationary point.
- A zero derivative is not always a local maximum or minimum.

Integrals, area and accumulation

■ How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
 - This lesson studies **antiderivative** : A function whose derivative equals the given integrand.
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■ The method

- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$.
 - A definite integral is signed accumulation.
 - Split at sign changes when total area or distance is required.
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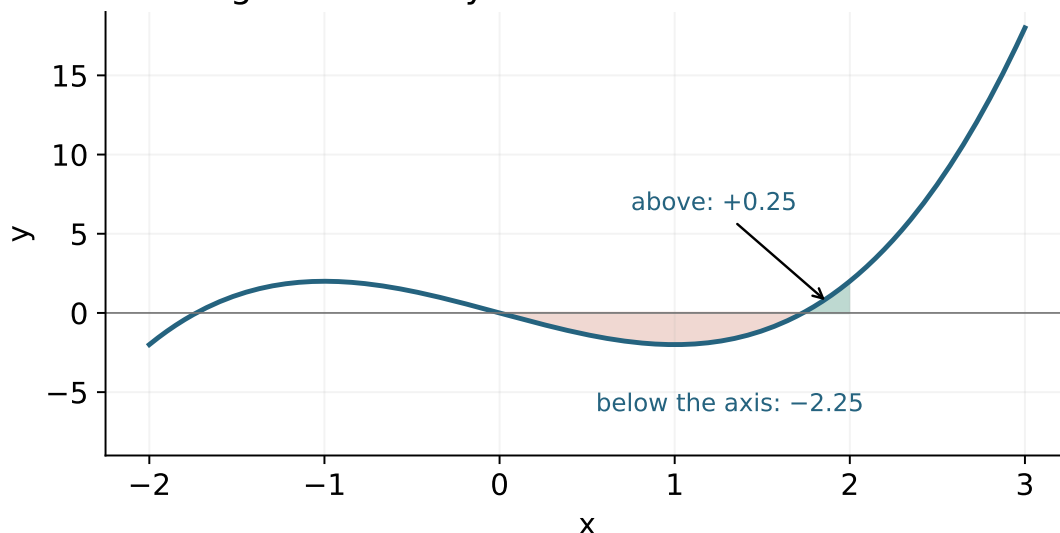
■ A worked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

Example:

For $v(t) = 3t^2 - 3$ over $0 \leq t \leq 2$, displacement = $[t^3 - 3t]_0^2 = 2$. Since v changes sign at $t = 1$, distance = $-[t^3 - 3t]_0^1 + [t^3 - 3t]_1^2 = 2 + 4 = 6$. The constants cancel only for a definite integral.

Signed area of $y = x^3 - 3x$ from 0 to 2 is -2



$$\int_0^2 (x^3 - 3x) dx = -2.25 + 0.25 = -2: \text{ area below the axis counts as negative}$$

■ Where it goes wrong

- The integral of $1/x$ is $\ln|x|+C$, not the power rule with $n=-1$.
- Signed area can be zero even when the total enclosed area is positive.

Not true: A definite integral always equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

■ Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Antiderivative:** A function whose derivative equals the given integrand.

- For $n \neq -1$, the integral of ax^n is $ax^{(n+1)}/(n+1)+C$.
- Watch for: the integral of $1/x$ is $\ln|x|+C$, not the power rule with $n=-1$.

■ **Write from a displacement of minus 3**

- Velocity is $v = 2t$ minus 4. Consider the motion from $t = 0$ to $t = 3$.
- Find the displacement and the total distance. A signed integral is not automatically the positive area.

■ **Write from a total distance of 5**

- The same motion: where does $v = 2t$ minus 4 change sign, and what is the total distance?
- At $t = 2$; the areas are 4 (below) and 1 (above), so distance $= 4 + 1 = 5$ against a displacement of minus 3.

■ **Write from a value of 16**

- Evaluate the definite integral of $6x$ squared from 0 to 2.
- The antiderivative $2x$ cubed gives 16 minus 0 – evaluate at both ends, then subtract.