

Geometry and trigonometry

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · SL

Scope and prerequisites

This handout covers selected SL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Angle reasoning, similarity and mensuration

■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

■ The method

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

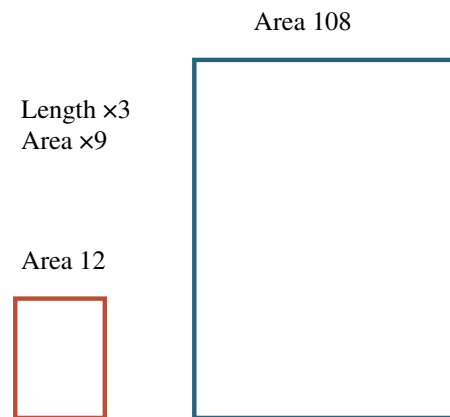
■ A worked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

Example:

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108 \text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216 \text{ cm}^3$. A cylinder with $r=3, h=5$ has volume $\pi r^2 h = 45\pi$.

A length scale factor is squared for area



Length scale factor 3 gives area scale factor $3^2 = 9$: $12 \times 9 = 108$

■ Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

Not true: Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

■ Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- Watch for: equal angles alone establish similarity, not equal size.

■ Write from a volume factor of 8

- Two similar solids have length scale factor 2.
- Find the area factor and the volume factor. Doubling lengths does not double the volume.

Right triangles and non-right triangles

■ Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.

■ The method

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite}/\text{hypotenuse}$, $\cos\theta = \text{adjacent}/\text{hypotenuse}$ and $\tan\theta = \text{opposite}/\text{adjacent}$.
- For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.

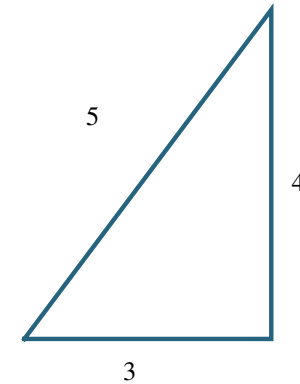
■ A worked case

$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Example:

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan\theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Label the sides first: here 3, 4 and hypotenuse 5, so $\sin = 4/5$, $\cos = 3/5$, $\tan = 4/3$ at the bottom-left angle

■ Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

■ Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
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Key:

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$.
- Watch for: label sides relative to the chosen angle.

■ Write from a hypotenuse of 13

- A right triangle has legs 5 and 12.
- Find the hypotenuse and the area. Pythagoras needs a right angle.

Radians, identities and trigonometric equations

■ How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
 - This lesson studies **radian** : The angle subtended by an arc whose length equals the radius.
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■ The method

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
 - Use $\sin^2\theta + \cos^2\theta = 1$ and angle identities to simplify or solve.
 - List every solution in the stated interval.
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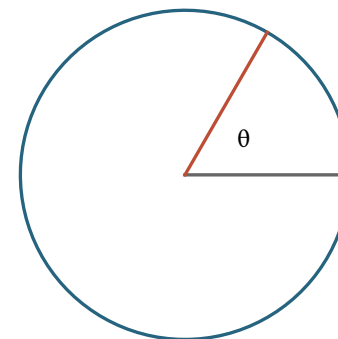
■ A worked case

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

Example:

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0 \leq x < 2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0 \leq 2x < 4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

$$\text{Arc length} = r\theta \text{ when } \theta \text{ is in radians}$$

■ Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.

Not true: The principal inverse sine value is always the only solution in a full turn. Sine takes most values twice in a full turn: $\sin x=0.5$ gives 30° and 150° .

■ Applying it

- Choose an identity that reduces the equation to one trigonometric function.
 - Keep exact familiar angles where possible, and check each solution against the original equation.
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■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Radian:** The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.

- Watch for: A calculator's inverse sine gives one principal value.

■ **Write from an arc length of 4**

- A sector has radius 10 and angle 0.4 radians.
- Find the arc length and the sector area. A principal inverse sine value is not the only solution in a full turn.

■ **Write from an arc of 3**

- A sector has radius 6 and angle 0.5 radians.
- Arc length $s = r \theta = 3$ – the radian definition doing its work.

■ **Write from a sector area of 12**

- A sector has radius 4 and angle 1.5 radians.
- Area = $r^2 \theta / 2 = 12$ – half the square of the radius scaled by the angle.