

# Geometry and trigonometry

International Baccalaureate · IB Diploma · Mathematics: Analysis  
and Approaches · SL

## Scope and prerequisites

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This handout covers selected SL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

## Angle reasoning, similarity and mensuration

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### ■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

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### ■ The method

- For similar shapes with length scale factor  $k$ , areas scale by  $k^2$  and volumes by  $k^3$ .
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

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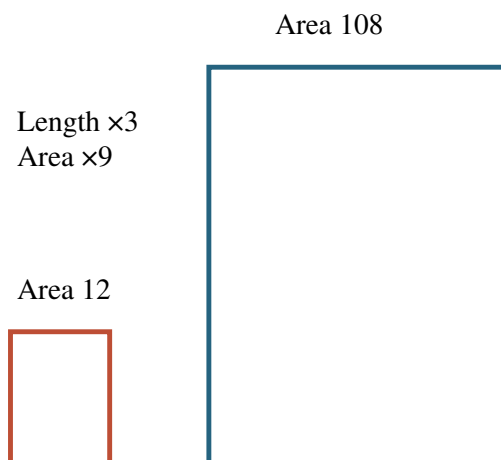
### ■ A worked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

### Example:

If model-to-real length factor is 3, a model area of  $12 \text{ cm}^2$  gives  $12 \times 3^2 = 108 \text{ cm}^2$  and a model volume of  $8 \text{ cm}^3$  gives  $8 \times 3^3 = 216 \text{ cm}^3$ . A cylinder with  $r=3, h=5$  has volume  $\pi r^2 h = 45\pi$ .

A length scale factor is squared for area



*Length scale factor 3 gives area scale factor  $3^2 = 9$ :  $12 \times 9 = 108$*

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### ■ Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

**Not true:** Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

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### ■ Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
  - A scale drawing is evidence only when the task permits measurement.
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### ■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and  $\pi$  exact and show each step. On calculator papers, still write the expression before its value.

**Key:**

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.

- For similar shapes with length scale factor  $k$ , areas scale by  $k^2$  and volumes by  $k^3$ .
- Watch for: equal angles alone establish similarity, not equal size.

### ■ Write from a volume factor of 8

- Two similar solids have length scale factor 2.
- Find the area factor and the volume factor. Doubling lengths does not double the volume.

## Right triangles and non-right triangles

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### ■ Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
  - This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.
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### ■ The method

- In a right triangle  $a^2 + b^2 = c^2$ ;  $\sin\theta = \text{opposite}/\text{hypotenuse}$ ,  $\cos\theta = \text{adjacent}/\text{hypotenuse}$  and  $\tan\theta = \text{opposite}/\text{adjacent}$ .
  - For other triangles, use the sine or cosine rule, or  $\text{area} = ab \sin C/2$ .
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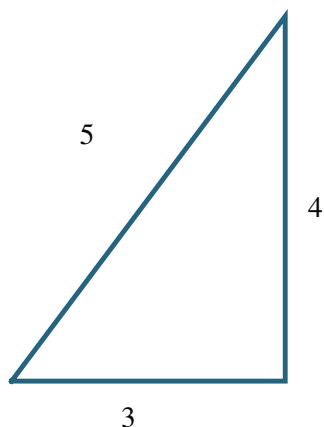
### ■ A worked case

$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

### Example:

The ladder length is  $c = \sqrt{3^2 + 4^2} = 5$  m. Its angle to the ground satisfies  $\tan\theta = 4/3$ , so  $\theta \approx 53.1^\circ$ . With two sides 6 and 8 enclosing  $60^\circ$ ,  $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$ .

## Label the known sides before choosing a rule



*Label the sides first: here 3, 4 and hypotenuse 5, so  $\sin = 4/5$ ,  $\cos = 3/5$ ,  $\tan = 4/3$  at the bottom-left angle*

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### ■ Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

**Not true:** Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

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### ■ Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
  - Explain why the chosen triangle contains the required length or angle.
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### ■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and  $\pi$  exact and show each step. On calculator papers, still write the expression before its value.

**Key:**

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- In a right triangle  $a^2 + b^2 = c^2$ ;  $\sin\theta = \text{opposite}/\text{hypotenuse}$ ,  $\cos\theta = \text{adjacent}/\text{hypotenuse}$  and  $\tan\theta = \text{opposite}/\text{adjacent}$ .
- Watch for: label sides relative to the chosen angle.

### ■ Write from a hypotenuse of 13

- A right triangle has legs 5 and 12.
- Find the hypotenuse and the area. Pythagoras needs a right angle.

## Radians, identities and trigonometric equations

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### ■ How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
  - This lesson studies **radian** : The angle subtended by an arc whose length equals the radius.
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### ■ The method

- For  $\theta$  in radians, arc length  $s = r\theta$  and sector area  $A = r^2\theta/2$ .
  - Use  $\sin^2\theta + \cos^2\theta = 1$  and angle identities to simplify or solve.
  - List every solution in the stated interval.
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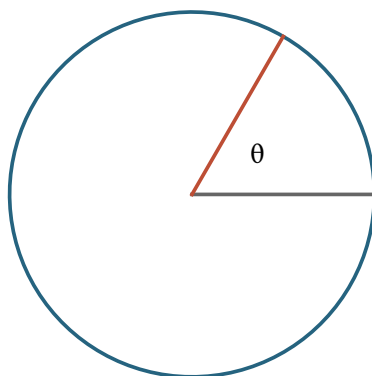
### ■ A worked case

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

### Example:

For  $r=4$  and  $\theta=\pi/3$ ,  $s=4\pi/3$  and  $A=8\pi/3$ . Solving  $\sin x=1/2$  on  $0 \leq x < 2\pi$  gives  $x=\pi/6$  and  $5\pi/6$ . For  $\sin 2x=1/2$ , solve first on  $0 \leq 2x < 4\pi$  before dividing by 2.

A radius fixes distance from the centre



radius  $\times$  angle gives arc length

*Arc length =  $r\theta$  when  $\theta$  is in radians*

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### ■ Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by  $\sin x$  can discard solutions where  $\sin x=0$ ; check that case first.

**Not true:** The principal inverse sine value is always the only solution in a full turn. Sine takes most values twice in a full turn:  $\sin x=0.5$  gives  $30^\circ$  and  $150^\circ$ .

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### ■ Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

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### ■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and  $\pi$  exact and show each step. On calculator papers, still write the expression before its value.

**Key:**

- **Radian:** The angle subtended by an arc whose length equals the radius.
- For  $\theta$  in radians, arc length  $s=r\theta$  and sector area  $A=r^2\theta/2$ .

- Watch for: A calculator's inverse sine gives one principal value.

■ **Write from an arc length of 4**

- A sector has radius 10 and angle 0.4 radians.
- Find the arc length and the sector area. A principal inverse sine value is not the only solution in a full turn.

■ **Write from an arc of 3**

- A sector has radius 6 and angle 0.5 radians.
- Arc length  $s = r\theta = 3$  – the radian definition doing its work.

■ **Write from a sector area of 12**

- A sector has radius 4 and angle 1.5 radians.
- Area =  $r^2 \theta \text{ over } 2 = 12$  – half the square of the radius scaled by the angle.