

Number and algebra

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · SL

Scope and prerequisites

This handout covers selected SL topics for the course first assessed in 2021; it is not the whole syllabus.

Prerequisites: read the stated quantities and units, and use arithmetic and the model conditions below.

The examples are original and fictional: they are not real measurements or real student investigations.

Exact arithmetic and estimation

■ Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

■ The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

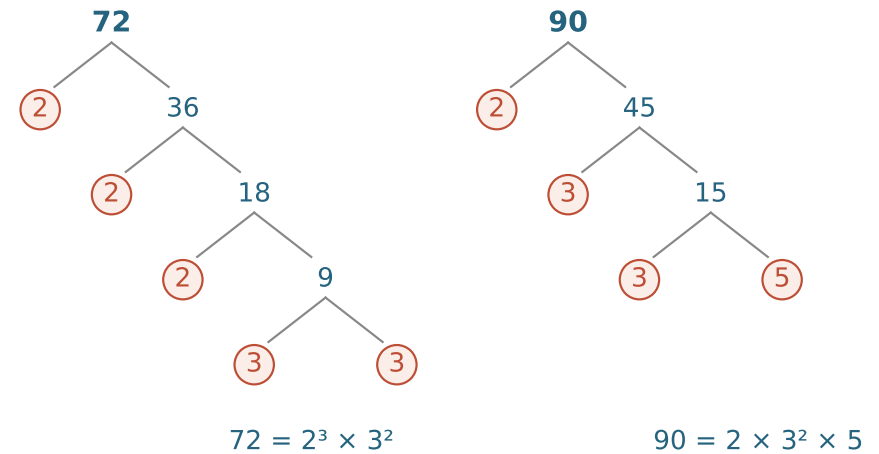
■ A worked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

Example:

$72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$. Their HCF is $2 \times 9 = 18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5 = 360$.

Factor trees: $\text{HCF}(72, 90) = 2 \times 3^2 = 18$



Factor trees: $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$, so $\text{HCF} = 2 \times 3^2 = 18$

■ Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

Not true: The HCF of two positive integers is always larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

■ Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.

Key:

- **Prime factor:** A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

■ Write from an HCF of 15

- Find the HCF and LCM of 45 and 75.
- The HCF is smaller than both integers.

Indices, surds and standard form

■ How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

■ The method

- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has $1 \leq a < 10$.

■ A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

Example:

0.000072 = 7.2×10^{-5} . Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

0.000072 = 7.2×10^{-5} : the point moves 5 places

5 jumps right

0 . 0 0 0 0 7 2

$$7.2 \times 10^{-5}$$

coefficient 7.2 is between 1 and 10; the power is -5

Moving the point 5 places right gives 7.2, so $0.000072 = 7.2 \times 10^{-5}$

■ Where it goes wrong

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .
- Do not round a surd when an exact answer is requested.

Not true: For every positive a , $a^2 + a^3$ equals a^5 . Index laws only add powers when the terms are multiplied: $2^2 + 2^3 = 12$, but $2^5 = 32$.

■ Applying it

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Index:** The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .

■ Write from 27 to the power 2/3

- Evaluate 27 to the power $2/3$.
- For a positive number, a squared plus a cubed is not a to the fifth.

Sequences, series and recurrence

■ Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** : The constant multiplier between consecutive terms of a geometric sequence.

■ The method

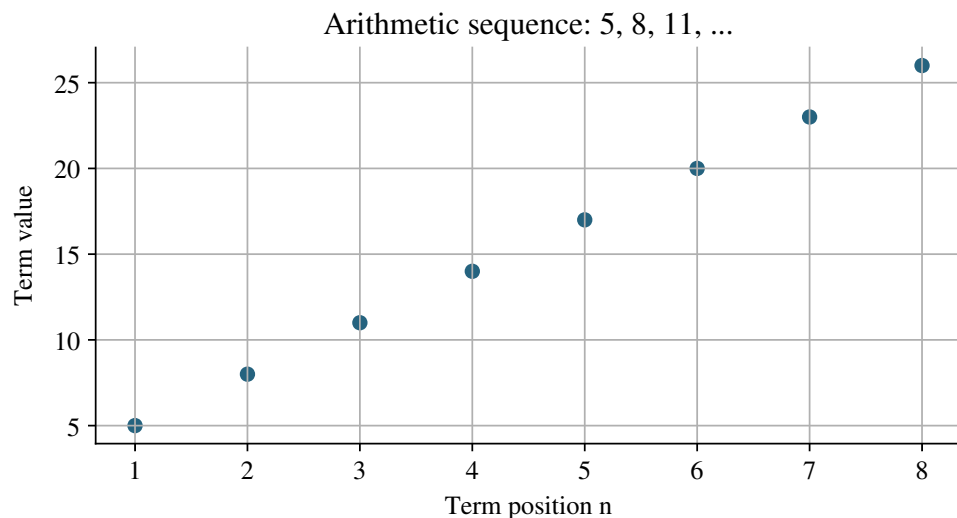
- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$.
- For a geometric progression, $u_n = ar^{n-1}$ and $S_n = a(1-r^n)/(1-r)$.
- An infinite geometric sum exists only if $|r| < 1$.

■ A worked case

$$u_n = a + (n-1)d, \quad S_n = \frac{n}{2} [2a + (n-1)d]$$

Example:

For $a=5, d=3, n=8$, $u_8 = 5 + 7 \times 3 = 26$ and $S_8 = 8(10+21)/2 = 124$. For $a=12, r=1/2$, $S_{\infty} = 12/(1-1/2) = 24$. For $u_{n+1} = 2u_n + 1$ with $u_1 = 1$, the next terms are 3, 7, 15.



$$u_n = 5 + 3(n-1): \text{ equal steps of 3 put the terms on a straight line}$$

■ Where it goes wrong

- The first term has index 1, so the exponent is $n-1$.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

Not true: Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1 < r < 1$; for $r=2$ the terms grow without limit.

■ Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Common ratio:** The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$.
- Watch for: the first term has index 1, so the exponent is $n-1$.

■ Write from the 6th term 29

- An arithmetic sequence has first term 4 and common difference 5.
- Find the 6th term. An infinite geometric series needs a common ratio with size below 1.

Percentages, ratio and proportional reasoning

■ Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier**: A factor that performs a percentage change in one multiplication.

■ The method

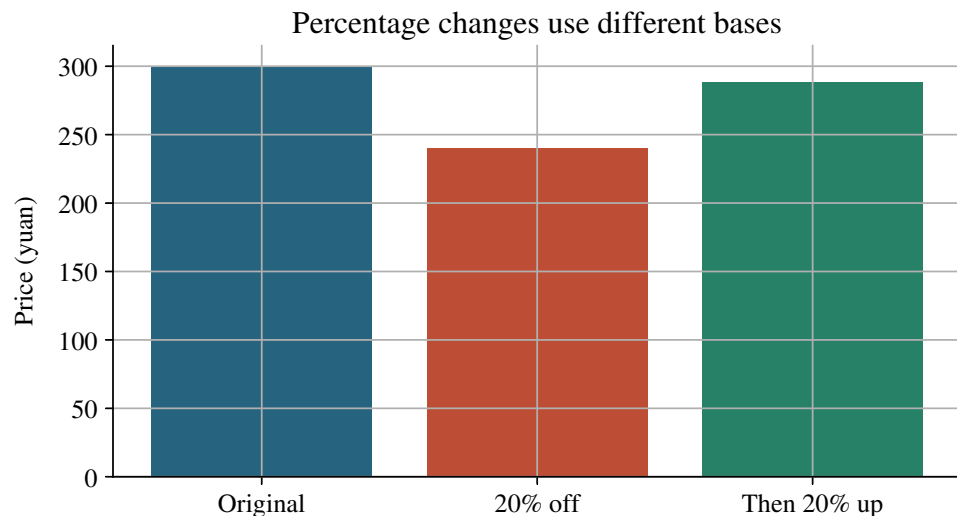
- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
- Reverse a percentage by dividing by the multiplier.
- In a ratio, first find the total number of parts.

■ A worked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

Example:

Let the original price be P. The model is sale price=0.8P. Hence P=240/0.8=300. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.



300 → 240 → 288 yuan: 20% off then 20% up does not return to 300, because the second 20% is of 240

■ Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

Not true: A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100 \times 0.8 \times 1.2 = 96$, not 100.

■ Applying it

- For direct proportion use $y=kx$; for inverse proportion use $y=k/x$.
- Calculate k from a known pair before using a new value.
- State what you held constant.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Multiplier:** A factor that performs a percentage change in one multiplication.
- A p% increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
- Watch for: A percentage uses a stated base.

■ Write from a final price of 192

- A price of 200 falls by 20 percent and then rises by 20 percent.
- Find the final price. The two changes do not cancel.