5.3 Chain, product and quotient rules; kinematics

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

KEY WORDS chain rule • product rule • quotient rule • velocity • acceleration • displacement

Objective

Build the skills to answer exam questions on **the chain, product and quotient rules** , and on **kinematics** .

You must be able to:

- use the chain rule for composite functions
- use the product and quotient rules
- use $v = \dfrac{ds}{dt}$ and $a = \dfrac{dv}{dt}$, and find displacement and distance from $v(t)$
- find maxima, minima and points of inflection of harder functions

1 Key knowledge and worked examples

■ **Chain, product and quotient rules; kinematics**

rule	for	derivative
chain	$y = f\big(g(x)\big)$	$f'\big(g(x)\big)\,g'(x)$
product	$y = uv$	$u'v + uv'$
quotient	$y = \dfrac{u}{v}$	$\dfrac{u'v - uv'}{v^2}$

Kinematics: displacement s, velocity $v = \dfrac{ds}{dt}$, acceleration $a = \dfrac{dv}{dt}$. The particle is **at rest** when $v = 0$. Displacement $= \int v\,dt$. **Distance travelled** $= \int_{t_1}^{t_2} |v|\,dt$: the particle changes direction where v changes sign.

Worked example. $y = x^2 \sin x$: $y' = \mathbf{2x}\sin\mathbf{x} + \mathbf{x^2}\cos\mathbf{x}$ (product rule).

Answer shapes: name u and v, write u' and v', then put them into the rule.

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2 Practice

2.1 Differentiate $y = (3x+1)^5$. [2]

$x \rightarrow$ [$3x+1$] $\rightarrow$ [$(\,)^5$]

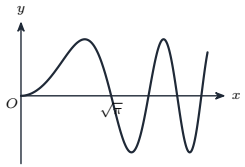
2.2 Differentiate $y = x^2 e^{3x}$. [3]

[$u = x^2$] $\times$ [$v = e^{3x}$]

2.3 Differentiate $y = \dfrac{\ln x}{x}$. [3]

[$u = \ln x$] / [$v = x$]

2.4 $y = \sin(x^2)$. Find $\dfrac{dy}{dx}$, and find the gradient at $x = \sqrt{\pi}$. [3]



3 Exam-style questions

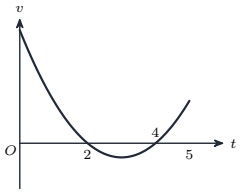
3.1 A particle moves in a straight line. Its velocity is $v = t^2 - 6t + 8$ m s^{-1} for $0 \leq t \leq 5$, and it starts at $s = 0$.

(a) Find the times when the particle is at rest. [2]

(b) Find the acceleration at $t = 1$. [2]

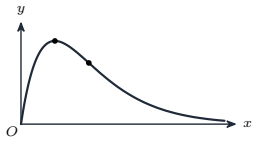
(c) Find the displacement at $t = 3$. [2]

(d) Find the total distance travelled in the first 5 seconds. [3]



3.2 $f(x) = xe^{-x}$ for $x \geq 0$.

(a) Show that $f'(x) = e^{-x}(1 - x)$. [2]



(b) Find the coordinates of the maximum point. [2]

(c) Show that $f''(x) = e^{-x}(x - 2)$. [2]

(d) Find the coordinates of the point of inflection. [2]

(e) Find the equation of the tangent at the origin. [2]

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