5.3 Chain, product and quotient rules; kinematics

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

KEY WORDS chain rule • product rule • quotient rule • velocity • acceleration • displacement

Objective

Build the skills to answer exam questions on **the chain, product and quotient rules** , and on **kinematics** .

You must be able to:

- use the chain rule for composite functions
- use the product and quotient rules
- use $v = \dfrac{ds}{dt}$ and $a = \dfrac{dv}{dt}$, and find displacement and distance from $v(t)$
- find maxima, minima and points of inflection of harder functions

1 Key knowledge and worked examples

■ **Chain, product and quotient rules; kinematics**

rule	for	derivative
chain	$y = f\big(g(x)\big)$	$f'\big(g(x)\big)\,g'(x)$
product	$y = uv$	$u'v + uv'$
quotient	$y = \dfrac{u}{v}$	$\dfrac{u'v - uv'}{v^2}$

Kinematics: displacement s, velocity $v = \dfrac{ds}{dt}$, acceleration $a = \dfrac{dv}{dt}$. The particle is **at rest** when $v = 0$. Displacement $= \int v\,dt$. **Distance travelled** $= \int_{t_1}^{t_2} |v|\,dt$: the particle changes direction where v changes sign.

Worked example. $y = x^2 \sin x$: $y' = \mathbf{2x}\sin\mathbf{x} + \mathbf{x^2}\cos\mathbf{x}$ (product rule).

Answer shapes: name u and v, write u' and v', then put them into the rule.

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2 Practice

2.1 Differentiate $y = (3x+1)^5$. [2]

$x \rightarrow$ [$3x+1$] $\rightarrow$ [$(\,)^5$]

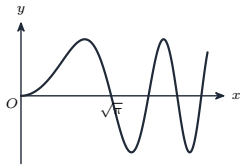
2.2 Differentiate $y = x^2 e^{3x}$. [3]

[$u = x^2$] $\times$ [$v = e^{3x}$]

2.3 Differentiate $y = \dfrac{\ln x}{x}$. [3]

[$u = \ln x$] / [$v = x$]

2.4 $y = \sin(x^2)$. Find $\dfrac{dy}{dx}$, and find the gradient at $x = \sqrt{\pi}$. [3]



3 Exam-style questions

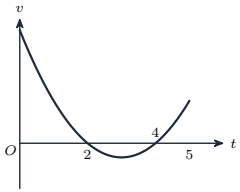
3.1 A particle moves in a straight line. Its velocity is $v = t^2 - 6t + 8$ m s^{-1} for $0 \leq t \leq 5$, and it starts at $s = 0$.

(a) Find the times when the particle is at rest. [2]

(b) Find the acceleration at $t = 1$. [2]

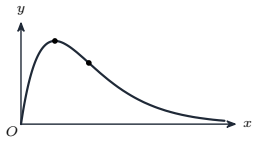
(c) Find the displacement at $t = 3$. [2]

(d) Find the total distance travelled in the first 5 seconds. [3]



3.2 $f(x) = xe^{-x}$ for $x \geq 0$.

(a) Show that $f'(x) = e^{-x}(1 - x)$. [2]



(b) Find the coordinates of the maximum point. [2]

(c) Show that $f''(x) = e^{-x}(x - 2)$. [2]

(d) Find the coordinates of the point of inflection. [2]

(e) Find the equation of the tangent at the origin. [2]

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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$5(3x+1)^4 \times 3 = \mathbf{15(3x+1)^4}$

2.2

$u' = 2x$, $v' = 3e^{3x}$; $2xe^{3x} + 3x^2e^{3x} = \mathbf{xe^{3x}(2+3x)}$

2.3

$$\frac{\frac{1}{x}\cdot x - \ln x \cdot 1}{x^2} = \frac{\mathbf{1 - \ln x}}{\mathbf{x^2}}$$

2.4

$2x\cos(x^2)$; $2\sqrt{\pi}\cos\pi = \mathbf{-2\sqrt{\pi}}$ (-3.54)

3.1

(a) $(t-2)(t-4) = 0$; $t = \mathbf{2}$ and $\mathbf{4}$ s

(b) $a = 2t - 6$; $a = \mathbf{-4}$ m s^{-2}

(c) $s = \frac{t^3}{3} - 3t^2 + 8t$; $s(3) = \mathbf{6}$ m

(d) $\int_0^5 |v|\,dt$; $\frac{20}{3} + \frac{4}{3} + \frac{4}{3} = \frac{\mathbf{28}}{\mathbf{3}}$ (9.33) m

3.2

(a) product rule: $1 \cdot e^{-x} + x \cdot (-e^{-x})$; $= e^{-x}(1-x)$

(b) $x = 1$; $\left(\mathbf{1}, \frac{\mathbf{1}}{\mathbf{e}}\right)$

(c) $-e^{-x}(1-x) + e^{-x}(-1)$; $= e^{-x}(x-2)$

(d) $x = 2$, and f'' changes sign there (R1); $\left(\mathbf{2}, \frac{\mathbf{2}}{\mathbf{e^2}}\right)$

(e) $f'(0) = 1$; $\mathbf{y = x}$

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