# 5.3 Chain, product and quotient rules; kinematics

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

**KEY WORDS** chain rule • product rule • quotient rule • velocity • acceleration • displacement

## Objective

Build the skills to answer exam questions on **the chain, product and quotient rules** and on **kinematics**.

**You must be able to:**

- use the chain rule for composite functions
- use the product and quotient rules
- use $v = \dfrac{ds}{dt}$ and $a = \dfrac{dv}{dt}$, and find displacement and distance from $v(t)$
- find maxima, minima and points of inflection of harder functions

## 1 Key knowledge and worked examples

### ■ Chain, product and quotient rules; kinematics

| rule | for | derivative |
|---|---|---|
| chain | $y = f\big(g(x)\big)$ | $f'\big(g(x)\big)\,g'(x)$ |
| product | $y = uv$ | $u'v + uv'$ |
| quotient | $y = \dfrac{u}{v}$ | $\dfrac{u'v - uv'}{v^2}$ |

**Kinematics:** displacement $s$, velocity $v = \dfrac{ds}{dt}$, acceleration $a = \dfrac{dv}{dt}$. The particle is **at rest** when $v = 0$. Displacement $= \int v\,dt$. **Distance travelled** $= \int_{t_1}^{t_2} |v|\,dt$: the particle changes direction where $v$ changes sign.

**Worked example.** $y = x^2 \sin x$: $y' = \mathbf{2x}\sin\mathbf{x} + \mathbf{x^2}\cos\mathbf{x}$ (product rule).

**Answer shapes:** name $u$ and $v$, write $u'$ and $v'$, then put them into the rule.



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## 2 Practice

**2.1** Differentiate $y = (3x + 1)^5$. [2]

$x \rightarrow$ `3x + 1` $\rightarrow$ `( )^5`

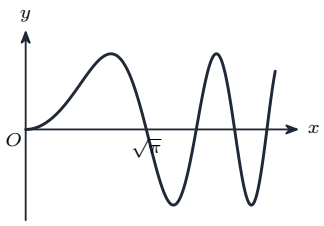
**2.2** Differentiate $y = x^2 e^{3x}$. [3]

`u = x^2` $\times$ `v = e^{3x}`

**2.3** Differentiate $y = \dfrac{\ln x}{x}$. [3]

`u = ln x` / `v = x`

**2.4** $y = \sin(x^2)$. Find $\dfrac{dy}{dx}$, and find the gradient at $x = \sqrt{\pi}$. [3]



## 3 Exam-style questions

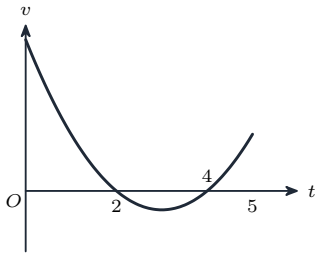
**3.1** A particle moves in a straight line. Its velocity is $v = t^2 - 6t + 8$ m s$^{-1}$ for $0 \leq t \leq 5$, and it starts at $s = 0$.

(a) Find the times when the particle is at rest. [2]

(b) Find the acceleration at $t = 1$. [2]

(c) Find the displacement at $t = 3$. [2]

(d) Find the total distance travelled in the first 5 seconds. [3]

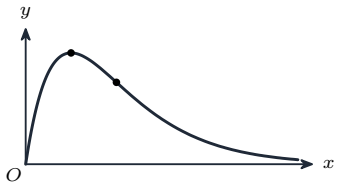





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**3.2** $f(x) = xe^{-x}$ for $x \geq 0$.

(a) Show that $f'(x) = e^{-x}(1 - x)$. [2]




(b) Find the coordinates of the maximum point. [2]

(c) Show that $f''(x) = e^{-x}(x - 2)$. [2]

(d) Find the coordinates of the point of inflection. [2]

(e) Find the equation of the tangent at the origin. [2]



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## Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

**2.1**

$5(3x+1)^4 \times 3 = \mathbf{15(3x+1)^4}$

**2.2**

$u' = 2x$, $v' = 3e^{3x}$; $2xe^{3x} + 3x^2e^{3x} = \mathbf{xe^{3x}(2+3x)}$

**2.3**

$$\frac{\frac{1}{x}\cdot x - \ln x \cdot 1}{x^2} = \frac{\mathbf{1 - \ln x}}{\mathbf{x^2}}$$

**2.4**

$2x\cos(x^2)$; $2\sqrt{\pi}\cos\pi = \mathbf{-2\sqrt{\pi}}$ $(-3.54)$

**3.1**

(a) $(t-2)(t-4) = 0$; $t = \mathbf{2}$ and $\mathbf{4}$ s

(b) $a = 2t - 6$; $a = \mathbf{-4}$ m s$^{-2}$

(c) $s = \frac{t^3}{3} - 3t^2 + 8t$; $s(3) = \mathbf{6}$ m

(d) $\int_0^5 |v|\,dt$; $\frac{20}{3} + \frac{4}{3} + \frac{4}{3} = \mathbf{\frac{28}{3}}$ $(9.33)$ m

**3.2**

(a) product rule: $1 \cdot e^{-x} + x \cdot (-e^{-x})$; $= e^{-x}(1-x)$

(b) $x = 1$; $\left(\mathbf{1}, \mathbf{\frac{1}{e}}\right)$

(c) $-e^{-x}(1-x) + e^{-x}(-1)$; $= e^{-x}(x-2)$

(d) $x = 2$, and $f''$ changes sign there (R1); $\left(\mathbf{2}, \mathbf{\frac{2}{e^2}}\right)$

(e) $f'(0) = 1$; $\mathbf{y = x}$



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