# 3.3 Radians, identities and trigonometric equations

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

**KEY WORDS** radian

## Objective

Radians, identities and trigonometric equations. Original classroom practice for IB SL; not an official IB paper.

## 1 Key knowledge and worked examples

### How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** : The angle subtended by an arc whose length equals the radius.

### The method

- For $\theta$ in radians, arc length s=r$\theta$ and sector area A=r$^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

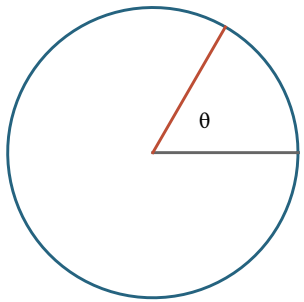
### A worked case

$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

**Example:**

For r=4 and $\theta=\pi/3$, s=$4\pi/3$ and A=$8\pi/3$. Solving sin x=1/2 on $0\leq x<2\pi$ gives x=$\pi/6$ and $5\pi/6$. For sin2x=1/2, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre




radius × angle gives arc length

### Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

**Not true:** The principal inverse sine value is always the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

### Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

### In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and $\pi$ exact and show each step. On calculator papers, still write the expression before its value.

**Key:**

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For $\theta$ in radians, arc length s=r$\theta$ and sector area A=r$^2\theta/2$.
- Watch for: A calculator's inverse sine gives one principal value.

■ **Write from an arc length of 4**

- A sector has radius 10 and angle 0.4 radians.
- Find the arc length and the sector area. A principal inverse sine value is not the only solution in a full turn.

## 2 Practice

**2.1** For r=4 and $\theta$=0.5 radians, find arc length. Show the relation, substitution and units. [2]

**2.2** For r=4 and $\theta$=0.5 radians, find sector area. Show the relation, substitution and units. [2]

**2.3** How many solutions does sin x=0.5 have on $0 \leq x < 2\pi$? Show the relation, substitution and units. [2]

**2.4** Decide whether this claim is true and justify: The principal inverse sine value is always the only solution in a full turn. [2]

**2.5** Solving sin x=1/2 on $0 \leq x < 2\pi$, why are there two solutions? Choose one answer.

- **A** Every equation has exactly two solutions
- **B** Sine is positive in two quadrants, so $\pi/6$ and $5\pi/6$ both work
- **C** The calculator inverse gives both values at once [1]

## 3 Exam-style questions

**3.1** Solve $2\sin^2 x - \sin x - 1 = 0$ for $0 \leq x < 2\pi$, showing why no solutions are lost. [4]

## Solutions

**2.1** Arc length=r$\theta$=4×0.5=2.

**2.2** Sector area=r$^2\theta$/2=16×0.5/2=4.

**2.3** The two solutions are $\pi/6$ and $5\pi/6$ in the stated interval.

**2.4** False. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

**2.5 B** The inverse sine returns only the principal value $\pi/6$. Sine is also positive in the second quadrant, giving $\pi$-$\pi/6$=$5\pi/6$. Both must be listed within the stated interval.

**3.1** Factor(2 sinx+1)(sinx−1)=0. sinx=1 gives$\pi/2$; sinx=−1/2 gives $7\pi/6$, $11\pi/6$. Both allowed roots are considered and the endpoint interval is respected.