3.2 Right triangles and non-right triangles

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS hypotenuse

Objective

Right triangles and non-right triangles. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.

The method

- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- For other triangles, use the sine or cosine rule, or area=ab sin C/2.

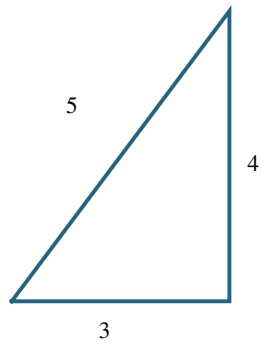
A worked case

$$a^2 + b^2 = c^2, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Example:

The ladder length is $c=\sqrt{(3^2+4^2)}=5$ m. Its angle to the ground satisfies $\tan\theta=4/3$, so $\theta\approx53.1°$. With two sides 6 and 8 enclosing 60°, $c^2=6^2+8^2-2\times6\times8\cos60°=52$.

Label the known sides before choosing a rule



Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Hypotenuse**: The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- Watch for: label sides relative to the chosen angle.

■ **Write from a hypotenuse of 13**

- A right triangle has legs 5 and 12.
- Find the hypotenuse and the area. Pythagoras needs a right angle.

2 Practice

2.1 Find the hypotenuse when the legs are 3 and 4. Show the relation, substitution and units. [2]

2.2 Find $\sin\theta$ when opposite=3 and hypotenuse=5. Show the relation, substitution and units. [2]

2.3 Two sides 6 and 8 enclose 90°. Find their area. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Pythagoras applies to every triangle, including triangles without a right angle. [2]

2.5 Which condition must hold before Pythagoras $a^2+b^2=c^2$ can be used? Choose one answer.

- **A** The triangle must contain a right angle
- **B** The triangle must be isosceles
- **C** All three sides must be known [1]

3 Exam-style questions

3.1 A triangle has sides a=7, b=10 and angle A=30°. Determine how many triangles are possible and give the possible B angles. [4]

Solutions

2.1 Pythagoras gives $\sqrt{(3^2+4^2)}=5$.

2.2 Sine=opposite/hypotenuse=3/5=0.6.

2.3 The sides are perpendicular, so area=$6\times8/2=24$.

2.4 False. Pythagoras needs a right angle; for other triangles use the cosine rule.

2.5 A Pythagoras is a property of right-angled triangles only, with c the hypotenuse. For other triangles the sine or cosine rule is needed.

3.1 Sine rule gives sinB=10 sin 30°/7=5/7. B≈45.585° or 134.415°; both leave a positive third angle. The ambiguous SSA case has two solutions, unlike a right-triangle shortcut.