2.3 Transformations of graphs and rational functions

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS translation • stretch • reflection • rational function • vertical asymptote • image

Objective

Build the skills to answer exam questions on **transformations of graphs and rational functions** .

You must be able to:

- apply translations, stretches and reflections to a graph, and describe them in words
- find the image of a point under one or more transformations
- find the asymptotes and intercepts of $y = \dfrac{ax+b}{cx+d}$ and sketch it

1 Key knowledge and worked examples

■ Transformations and rational functions

new graph	**transformation of $y = f(x)$**
$y = f(x) + b$	translation b up
$y = f(x - a)$	translation a to the right
$y = pf(x)$	vertical stretch, scale factor p
$y = f(qx)$	horizontal stretch, scale factor $\frac{1}{q}$
$y = -f(x)$ / $y = f(-x)$	reflection in the x-axis / y-axis

Changes **inside** the bracket act on x and work "the opposite way".

Rational function $y = \dfrac{ax+b}{cx+d}$: vertical asymptote $x = -\dfrac{d}{c}$ (bottom $= 0$), horizontal asymptote $y = \dfrac{a}{c}$, x-intercept where the top $= 0$, y-intercept $\dfrac{b}{d}$.

Worked example 1. The point $(4, 5)$ on $y = f(x)$ moves under $y = 2f(x-1) + 3$ to $(4+1,\ 2\times 5+3) = (\mathbf{5}, \mathbf{13})$.

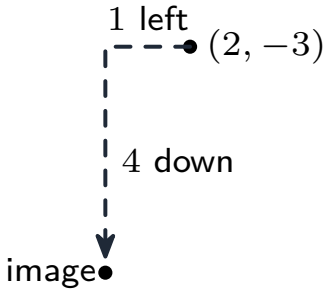
exercises.lol exercises.com.cn

Worked example 2. $y = \dfrac{2x+1}{x-3}$: asymptotes $x = \mathbf{3}$ and $y = \mathbf{2}$; intercepts $\left(-\frac{1}{2}, 0\right)$ and $\left(0, -\frac{1}{3}\right)$.

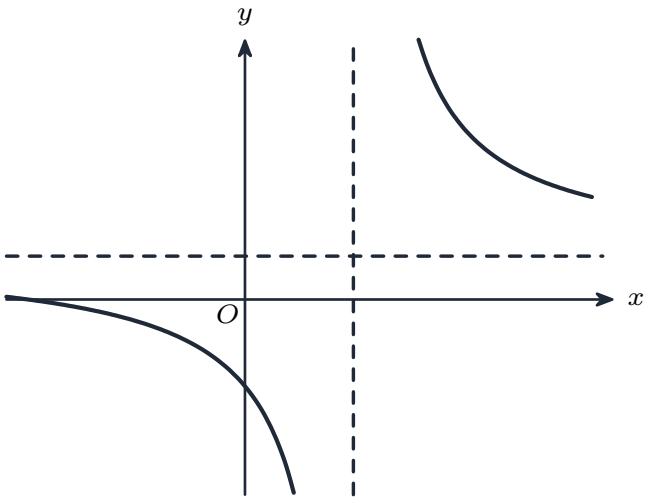
Answer shapes: name each transformation with its size and direction ("vertical stretch, scale factor 3").

2 Practice

2.1 The point $(2, -3)$ lies on $y = f(x)$. Find its image on the graph of $y = f(x+1)-4$. [2]

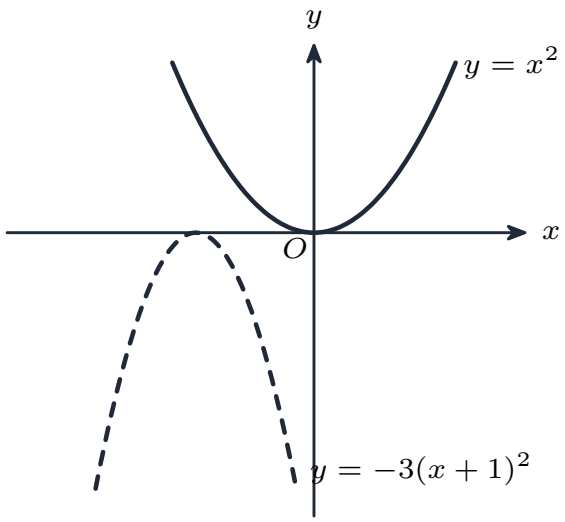


2.2 For $y = \dfrac{x+4}{x-2}$, write down the equations of the asymptotes, and find the intercepts with the axes. [3]



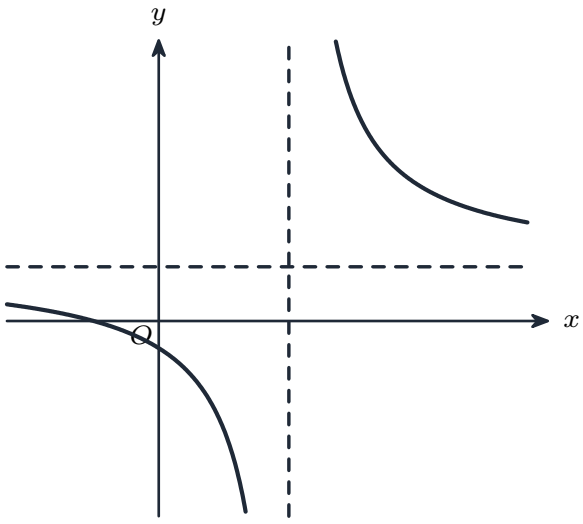
exercises.lol exercises.com.cn

2.3 Describe a sequence of transformations that maps the graph of $y = x^2$ onto the graph of $y = -3(x+1)^2$. [3]



3 Exam-style questions

3.1 $f(x) = \dfrac{3}{x-2} + 1,\ x \neq 2.$



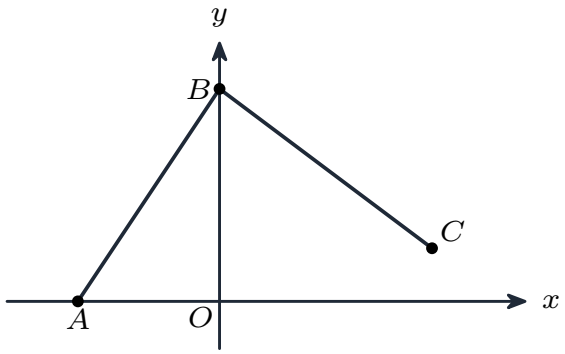
(a) Write down the equations of the asymptotes. [2]

exercises.lol exercises.com.cn

(b) Find the coordinates of the intercepts with the axes. [2]

(c) Describe the transformations that map $y = \frac{1}{x}$ onto $y = f(x)$. [3]

3.2 The graph of $y = f(x)$, $-2 \leq x \leq 3$, consists of two straight line segments joining $A(-2, 0)$, $B(0, 4)$ and $C(3, 1)$.



(a) Write down the image of B on the graph of $y = f(x) - 3$. [1]

(b) Write down the image of C on the graph of $y = f(2x)$. [1]

(c) Find the images of A, B and C on the graph of $y = -f(x + 1)$. [3]

(d) Write down the range of $y = -f(x + 1)$. [1]

exercises.lol exercises.com.cn

(e) Find the range of $y = 2f(x) + 1$. [2]

exercises.lol exercises.com.cn

Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

x: $2 - 1 = 1$; y: $-3 - 4 = -7$, so $(\mathbf{1}, \mathbf{-7})$

2.2

$x = 2$ and $y = 1$; $(-4, 0)$; $(0, -2)$

2.3

translation 1 to the left; vertical stretch, scale factor 3; reflection in the x-axis

3.1

(a) $x = 2$; $y = 1$

(b) $\dfrac{3}{x-2} = -1$, $(-1, 0)$; $\left(0, -\frac{1}{2}\right)$

(c) vertical stretch, scale factor 3; translation 2 to the right; translation 1 up

3.2

(a) $(0, 1)$

(b) $(1.5, 1)$

(c) $A'(-3, 0)$; $B'(-1, -4)$; $C'(2, -1)$

(d) $-4 \leq y \leq 0$

(e) range of f is $0 \leq f(x) \leq 4$; $1 \leq y \leq 9$