1.3 Sequences, series and recurrence

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS common ratio

Objective

Sequences, series and recurrence. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** : The constant multiplier between consecutive terms of a geometric sequence.

The method

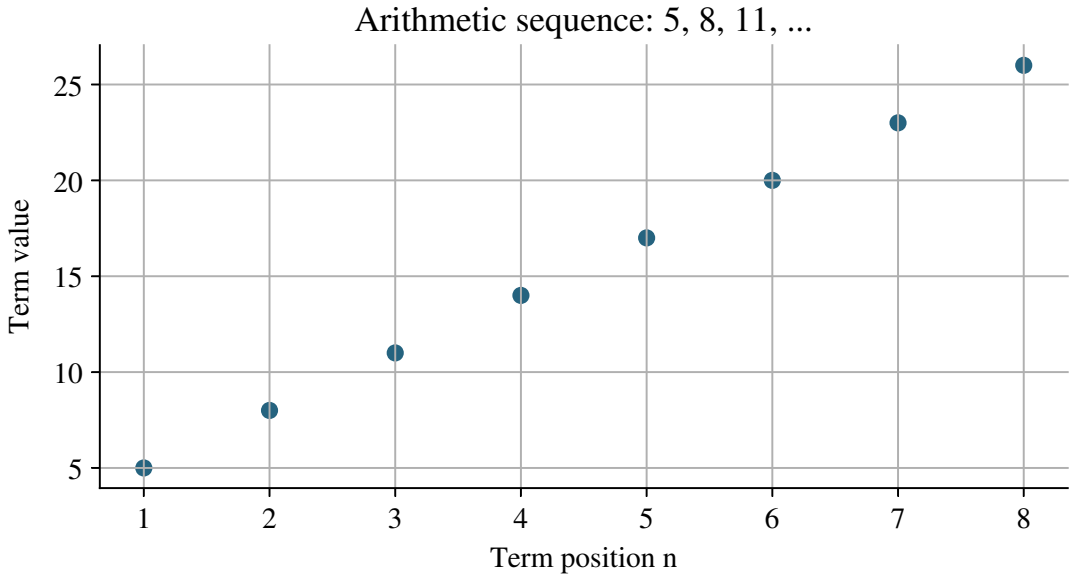
- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- For a geometric progression, u_n=ar^(n-1) and S_n=a(1-r^n)/(1-r).
- An infinite geometric sum exists only if |r|<1.

A worked case

$$u_n = a + (n-1)d, \qquad S_n = \frac{n}{2}\,[2a + (n-1)d]$$

Example:

For a=5,d=3,n=8, u_8=5+7×3=26 and S_8=8(10+21)/2=124. For a=12,r=1/2, S infinity=12/(1-1/2)=24. For u_(n+1)=2u_n+1 with u_1=1, the next terms are 3,7,15.



Where it goes wrong

- The first term has index 1, so the exponent is n-1.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

Not true: Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1<r<1$; for $r=2$ the terms grow without limit.

Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Common ratio**: The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, $u_n=a+(n-1)d$ and $S_n=n(2a+(n-1)d)/2$.
- Watch for: the first term has index 1, so the exponent is n-1.

■ **Write from the 6th term 29**

- An arithmetic sequence has first term 4 and common difference 5.
- Find the 6th term. An infinite geometric series needs a common ratio with size below 1.

2 Practice

2.1 For a=5,d=3, find u_8. Show the relation, substitution and units. [2]

2.2 Find the sum of the first 8 terms with a=5,d=3. Show the relation, substitution and units. [2]

2.3 Find the infinite sum with a=12,r=0.5. Show the relation, substitution and units.[2]

2.4 Decide whether this claim is true and justify: Common ratio means the constant multiplier between consecutive terms of a geometric sequence. [2]

2.5 An infinite geometric series has a finite sum only when which condition holds? Choose one answer.

- **A** The first term is positive
- **B** The common ratio satisfies |r|<1
- **C** The common ratio is a whole number [1]

3 Exam-style questions

3.1 A sequence obeys $u_1=4$ and $u_{n+1}=0.5\ u_n+3$. Find its fixed point and express u_n-6 as a geometric sequence. Does the sum of u_n converge? [4]

Solutions

2.1 $u_8=5+(8-1)\times3=26$.

2.2 $S_8=8[2\times5+7\times3]/2=124$.

2.3 Since $|r|<1$, $S\infty=12/(1-0.5)=24$.

2.4 True. True: The constant multiplier between consecutive terms of a geometric sequence.

2.5 B The terms ar^(n-1) shrink towards zero only when $|r|<1$, so the sum converges to a/(1-r). With $|r|\geq1$ the terms do not shrink and the sum diverges.

3.1 Fixed point 6. Subtracting 6 gives $u_{n+1}-6=0.5(u_n-6)$, so $u_n=6-2(0.5)$^(n−1). Terms approach 6, so their infinite sum diverges; a convergent sequence need not give a convergent series.