

Exercise walk-through

6.1

Planning a mathematical exploration

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

6.1 Planning a mathematical exploration

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS assumption

Objective

Planning a mathematical exploration. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

■ What makes an investigation mathematical?

- A student wants to predict queue waiting time at the school canteen. A useful exploration needs a question, data and mathematics the student can explain.
- This lesson studies **assumption** : A condition adopted to make a mathematical

model possible and whose effect should be evaluated.

■ The method

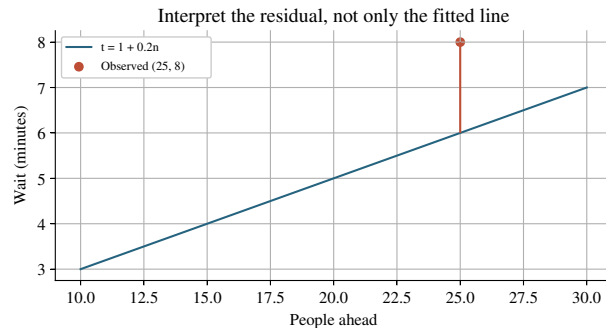
- Frame a focused question with measurable variables and a feasible method.
 - Keep a dated record of decisions, sources and mathematical reasoning.
 - Use technology to support analysis, and explain every important output in your own words.
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■ A worked case

$$\hat{t} = 1 + 0.2n, \quad e = t - \hat{t}$$

Example:

A pilot records mean waiting times of 3,5,7 minutes when 10,20,30 people are ahead. A first model is $t=1+0.2n$. At $n=25$ it predicts 6 minutes. An actual wait of 8 gives residual 2; investigate varying service rates rather than hiding the mismatch.



■ Where it goes wrong

- Do not copy a sample exploration, invent data or manufacture personal engagement.
- A prediction is not evidence until it is compared with observations.
- Consent and privacy matter when collecting school data.

Not true: The school-supervised mathematical exploration can be replaced by a timed written mock exam. The exploration is an extended piece of written work assessed on its own criteria; a timed exam tests something different.

■ Applying it

- The school supervises topic approval, checkpoints, authenticity and assessment.
- For the current 2021 course use the current school's guide and criteria; the 2029 inquiry criteria belong to a different assessment version.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Assumption:** A condition adopted to make a mathematical model possible and whose effect should be evaluated.
 - Frame a focused question with measurable variables and a feasible method.
 - Watch for: do not copy a sample exploration, invent data or manufacture personal engagement.
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■ Write from a predicted wait of 5

- A model says $\text{wait} = 2 \text{ plus } 0.3n$. At $n = 10$ the observed wait is 8.
- Find the prediction and the residual. A timed mock does not replace the exploration.

Exercises 2.1·2.3

2.1 For $t=1+0.2n$, find predicted wait at $n=25$. Show the relation, substitution and units. [2]

2.2 Find the residual for observed 8 and predicted 6. Show the relation, substitution and units. [2]

2.3 Three independent measurements are 4,6,8. Find their mean. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For $t=1+0.2n$, find predicted wait at $n=25$. Show the relation, substitution and units. [2]

2.1 Use $n=25$ in the model: $1+0.2\times 25=6$ minutes.

2.2 Find the residual for observed 8 and predicted 6. Show the relation, substitution and units. [2]

2.2 Residual= $8-6=2$ minutes.

2.3 Three independent measurements are 4,6,8. Find their mean. Show the relation, substitution and units. [2]

2.3 Mean= $(4+6+8)/3=6$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Assumption means a condition adopted to make a mathematical model possible and whose effect should be evaluated. [2]

2.5 A pilot study gives $t=1+0.2n$, but an actual wait of 8 minutes at $n=25$ gives residual 2. What is the correct response? Choose one answer.

- **A** Delete the observation and refit the model without mentioning it
- **B** Hide the mismatch, since the model fitted every other observation
- **C** Investigate why - for example a varying service rate - and record the reasoning [1]

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Assumption means a condition adopted to make a mathematical model possible and whose effect should be evaluated. [2]

2.4 True. True: A condition adopted to make a mathematical model possible and whose effect should be evaluated.

2.5 A pilot study gives $t=1+0.2n$, but an actual wait of 8 minutes at $n=25$ gives residual 2. What is the correct response? Choose one answer.

- **A** Delete the observation and refit the model without mentioning it
- **B** Hide the mismatch, since the model fitted every other observation
- **C** Investigate why - for example a varying service rate - and record the reasoning [1]

2.5 C A residual is evidence about the model's limits. Investigating a better assumption, with dated records, is exactly the mathematical thinking the exploration rewards; hiding it is a failure of authenticity.

Exercise 3.1

3.1 Fictional wait times for $n=10, 20, 30$ are 4, 7, 11 minutes. A proposed model is $t=1+0.3 n$. Find residuals and propose a useful next investigation; do not write a learner's assessed exploration. [4]

Exercise 3.1

3.1 Fictional wait times for $n=10, 20, 30$ are 4, 7, 11 minutes. A proposed model is $t=1+0.3 n$. Find residuals and propose a useful next investigation; do not write a learner's assessed exploration. [4]

3.1 Predictions 4, 7, 10; residuals 0, 0, 1 minute. Collect repeated observations across a wider justified range, inspect curvature/variation and compare models. One residual pattern from three invented values does not validate causality or certify an IB exploration.