Exercise walk-through

5.1
Derivatives and stationary points

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

5.1 Derivatives and stationary points

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS derivative

Objective

Derivatives and stationary points. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

■ What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.

■ The method

- For y=ax^n, dy/dx=anx^(n-1).
- A stationary point satisfies dy/dx=0.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

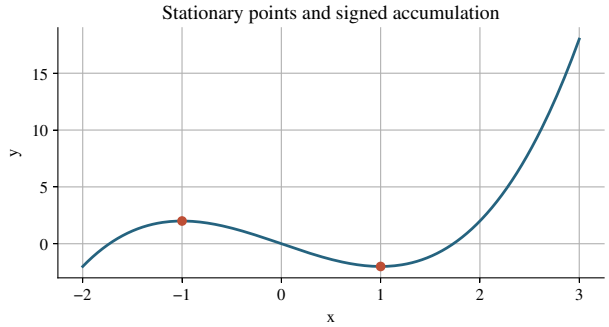
A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \qquad f'(x) = 0$$

Example:

For $y=x^3-3x$, $dy/dx=3x^2-3$. At x=1, the gradient is 0 and y=-2. The second derivative is 6x, positive at x=1, so this is a local minimum. At x=-1, y=2 and the second derivative

is negative, giving a local maximum.



■ **Where it goes wrong**

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

Not true: Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

■ **Applying it**

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.

■ **In your course**

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For y=ax^n, dy/dx=anx^(n-1).
- Watch for: A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.

Write from a derivative of 6

- For y = x squared minus 4x, find the derivative at x = 5 and the y-coordinate of the stationary point.
- A zero derivative is not always a local maximum or minimum.

Exercises 2.1·2.3

2.1 For $y=x^3-3x$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

2.2 Find the y-coordinate of the stationary point at $x=1$. Show the relation, substitution and units. [2]

2.3 For $y=4x^3$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For $y=x^3-3x$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

2.1 $dy/dx=3x^2-3$. At $x=2$ it is $12-3=9$.

2.2 Find the y-coordinate of the stationary point at $x=1$. Show the relation, substitution and units. [2]

2.2 Substitute into the original curve: $y(1)=1-3=-2$.

2.3 For $y=4x^3$, find dy/dx at $x=2$. Show the relation, substitution and units. [2]

2.3 The derivative is $12x^2$; at 2 it is $12\times4=48$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Derivative means the instantaneous rate of change, also the gradient of a tangent. [2]

2.5 At $x=0$ the curve $y=x^3$ has $dy/dx=0$. Why is it not a local maximum or minimum? Choose one answer.

- **A** The gradient does not change sign: the curve keeps increasing through the point
- **B** Every stationary point is a maximum or minimum; $x=0$ is a minimum of x^3
- **C** The derivative $3x^2$ is undefined at $x=0$, so the point is not stationary [1]

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Derivative means the instantaneous rate of change, also the gradient of a tangent. [2]

2.4 True. True: The instantaneous rate of change, also the gradient of a tangent.

2.5 At $x=0$ the curve $y=x^3$ has $dy/dx=0$. Why is it not a local maximum or minimum? Choose one answer.

- **A** The gradient does not change sign: the curve keeps increasing through the point
- **B** Every stationary point is a maximum or minimum; $x=0$ is a minimum of x^3
- **C** The derivative $3x^2$ is undefined at $x=0$, so the point is not stationary [1]

2.5 A The derivative $3x^2$ is positive on both sides of $x=0$, so the point is stationary but the function continues to rise - a point of inflection. Classification needs the sign change or the second derivative, not merely $dy/dx=0$.

Exercise 3.1

3.1 For $y=x^3-3x$ on $[-2, 2]$, find the absolute maximum and minimum, including all candidates. [4]

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3.1 Derivative $3x^2-3=0$ at ± 1. Values $y(-2)=-2$, $y(-1)=2$, $y(1)=-2$, $y(2)=2$. Maximum 2 at -1, 2; minimum -2 at -2, 1. Checking only stationary points misses endpoint ties.