Exercise walk-through

3.1
Angle reasoning, similarity and mensuration

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

3.1 Angle reasoning, similarity and mensuration

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS scale factor

Objective

Angle reasoning, similarity and mensuration. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding

lengths in similar shapes.

The method

- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

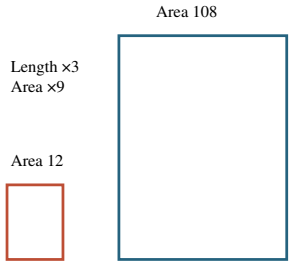
A worked case

$$\frac{A_2}{A_1} = k^2, \qquad \frac{V_2}{V_1} = k^3$$

Example:

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108$ cm^2 and a model volume of 8 cm^3 gives $8 \times 3^3 = 216$ cm^3. A cylinder with $r=3, h=5$ has volume $\pi r^2 h = 45\pi$.

A length scale factor is squared for area



■ **Where it goes wrong**

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

Not true: Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

■ **Applying it**

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

■ **In your course**

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Scale factor**: The multiplier that relates corresponding lengths in similar shapes.
- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- Watch for: equal angles alone establish similarity, not equal size.

- **Write from a volume factor of 8**
  - Two similar solids have length scale factor 2.
  - Find the area factor and the volume factor. Doubling lengths does not double the volume.

Exercises 2.1·2.3

2.1 Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

2.2 Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

2.3 Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

2.1 Area scales by the square of the length factor: $3^2=9$.

2.2 Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

2.2 Volume scales by the cube: $3^3=27$.

2.3 Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

2.3 Interior-angle sum$=(5-2)\times180=540°$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Scale factor means the multiplier that relates corresponding lengths in similar shapes. [2]

2.5 Two similar solids have length scale factor 3. Why is the volume factor 27? Choose one answer.

- **A** Volume scales with the square of the length factor: $3^2=9$, then add the base
- **B** Volume scales with the cube of the length factor: $3^3=27$
- **C** Volume scales with the length factor itself: a factor of 3 gives 3 [1]

3.1 Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: Scale factor means the multiplier that relates corresponding lengths in similar shapes. [2]

2.4 True. True: The multiplier that relates corresponding lengths in similar shapes.

2.5 Two similar solids have length scale factor 3. Why is the volume factor 27? Choose one answer.

- **A** Volume scales with the square of the length factor: $3^2=9$, then add the base
- **B** Volume scales with the cube of the length factor: $3^3=27$
- **C** Volume scales with the length factor itself: a factor of 3 gives 3 [1]

2.5 B Each of the three dimensions multiplies by 3, so volume multiplies by $3\times3\times3=27$. Area uses the square and volume the cube of the length factor.

3.1 Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

3.1 Length ratio 3:2, area 9:4. Area scales as the square and volume as the cube of the length factor, assuming genuine geometric similarity.