

Exercise walk-through

2.1

Domains, inverses and composition

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

2.1 Domains, inverses and composition

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS domain

Objective

Domains, inverses and composition. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

■ Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
 - This lesson studies **domain** : The set of allowed inputs to a function.
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■ The method

- State the domain and range.
 - For an inverse, first ensure the function is one-to-one on its domain.
 - Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .
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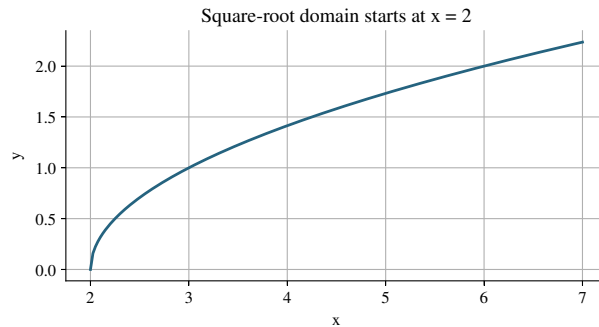
■ A worked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

Example:

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus f in-

verse(x)=x²+2 with x≥0. For g(x)=x+3, fg(1)=f(4)=√2.



■ Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

Not true: Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one ($f(1)=f(-1)$ for x^2), so its domain must be restricted first.

■ Applying it

- Check $f(f^{-1}(x))=x$ on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

■ In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Domain:** The set of allowed inputs to a function.
 - State the domain and range.
 - Watch for: squaring can introduce extraneous solutions.
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■ Write from an inverse value of 5

- For $f(x)=3x-1$, find the inverse image of 14.
- A quadratic on all real numbers is not one-to-one, so it needs a restricted domain before it has an inverse.

Exercises 2.1·2.3

2.1 For $f(x)=2x+3$, find $f^{-1}(11)$. Show the relation, substitution and units. [2]

2.2 For $f(x)=2x+3$ and $g(x)=x^2$, find $fg(4)$. Show the relation, substitution and units. [2]

2.3 For $f(x)=\sqrt{x-2}$, find $f(11)$. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 For $f(x)=2x+3$, find $f^{-1}(11)$. Show the relation, substitution and units. [2]

2.1 Solve $2x+3=11$, giving $x=4$.

2.2 For $f(x)=2x+3$ and $g(x)=x^2$, find $fg(4)$. Show the relation, substitution and units. [2]

2.2 Apply g first: $g(4)=16$; then $f(16)=2 \times 16 + 3 = 35$.

2.3 For $f(x)=\sqrt{x-2}$, find $f(11)$. Show the relation, substitution and units. [2]

2.3 Substitute into the square root: $\sqrt{11-2}=\sqrt{9}=3$.

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every quadratic function on all real numbers has an inverse function. [2]

2.5 Why must $f(x)=x^2$ have its domain restricted before an inverse is defined? Choose one answer.

- **A** Two different inputs share each output, so the reverse is not a function
- **B** The range of x^2 is empty unless the domain is restricted to $x \geq 0$
- **C** Squaring is undefined for negative inputs, so those inputs must be removed first [1]

Exercises 2.4·2.5

2.4 Decide whether this claim is true and justify: Every quadratic function on all real numbers has an inverse function. [2]

2.4 False. A quadratic on all real numbers is many-to-one ($f(1)=f(-1)$ for x^2), so its domain must be restricted first.

2.5 Why must $f(x)=x^2$ have its domain restricted before an inverse is defined? Choose one answer.

- **A** Two different inputs share each output, so the reverse is not a function
- **B** The range of x^2 is empty unless the domain is restricted to $x \geq 0$
- **C** Squaring is undefined for negative inputs, so those inputs must be removed first [1]

2.5 A x^2 sends both 3 and -3 to 9, so an unrestricted inverse could not choose a single value. Restricting to $x \geq 0$ makes the function one-to-one and the inverse $\sqrt{x}$ well defined.

Exercise 3.1

3.1 For $f(x)=x^2$ on $x \geq 0$ and $g(x)=x-3$, find $f(g(x))$ and its domain, then find f inverse(16).
Explain why the restriction matters. [4]

Exercise 3.1

3.1 For $f(x)=x^2$ on $x \geq 0$ and $g(x)=x-3$, find $f(g(x))$ and its domain, then find f inverse(16). Explain why the restriction matters. [4]

3.1 Composite $= (x-3)^2$ with $x \geq 3$ because $g(x)$ must lie in f 's domain. Inverse $16=4$. Without restriction x^2 is not one-to-one and the inverse would not be a function on the full real domain.