

Exercise walk-through

1.2

Indices, surds and standard form

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

1.2 Indices, surds and standard form

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS index

Objective

Indices, surds and standard form. Original classroom practice for IB SL; not an official IB paper.

1 Key knowledge and worked examples

■ How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
 - This lesson studies **index** : The power to which a base is raised.
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■ The method

- For the same positive base, multiplication adds indices and division subtracts them.
 - A negative index means reciprocal; a fractional index represents a root.
 - Standard form has $1 \leq a < 10$.
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■ A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a} \right)^m$$

Example:

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$.

$$0.000072 = 7.2 \times 10^{-5}$$

7.2 is the coefficient; -5 is the power

The coefficient is between 1 and 10

■ **Where it goes wrong**

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .
- Do not round a surd when an exact answer is requested.

Not true: For every positive a , $a^2 + a^3$ equals a^5 . Index laws only add powers when the terms are multiplied: $2^2 + 2^3 = 12$, but $2^5 = 32$.

■ **Applying it**

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

■ **In your course**

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key:

- **Index:** The power to which a base is raised.
 - For the same positive base, multiplication adds indices and division subtracts them.
 - Watch for: index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .
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■ Write from 27 to the power $2/3$

- Evaluate 27 to the power $2/3$.
- For a positive number, a squared plus a cubed is not a to the fifth.

Exercises 2.1-2.3

2.1 Evaluate $16^{(3/4)}$. Show the relation, substitution and units. [2]

2.2 Evaluate $2^{(-3)}$. Show the relation, substitution and units. [2]

2.3 Write $7.2 \times 10^{(-5)}$ as a decimal. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Evaluate $16^{(3/4)}$. Show the relation, substitution and units. [2]

2.1 $16=2^4$, so $16^{(3/4)}=2^3=8$.

2.2 Evaluate $2^{(-3)}$. Show the relation, substitution and units. [2]

2.2 A negative index gives a reciprocal: $2^{(-3)}=1/8=0.125$.

2.3 Write $7.2 \times 10^{(-5)}$ as a decimal. Show the relation, substitution and units. [2]

2.3 Move the decimal point five places left: $7.2 \times 10^{(-5)}=0.000072$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: For every positive a , $a^2 + a^3$ equals a^5 . [2]

2.5 Why is $16^{3/4}$ equal to 8? Choose one answer.

- **A** The numerator cube applies after the fourth root: $(16^{1/4})^3 = 2^3$
 - **B** Multiply the base by the index first: $16 \times 3/4 = 12$, then round to 8
 - **C** The fourth root applies after cubing: $16^3 = 4096$, and $\sqrt[4]{4096}$ is 8
- [1]

3.1 Solve $x^{3/2} = 27$ over the real domain used for this power. Check the result and explain why a negative candidate is invalid. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: For every positive a , $a^2 + a^3$ equals a^5 . [2]

2.4 False. Index laws only add powers when the terms are multiplied: $2^2 + 2^3 = 12$, but $2^5 = 32$.

2.5 Why is $16^{3/4}$ equal to 8? Choose one answer.

- **A** The numerator cube applies after the fourth root: $(16^{1/4})^3 = 2^3$
- **B** Multiply the base by the index first: $16 \times 3/4 = 12$, then round to 8
- **C** The fourth root applies after cubing: $16^3 = 4096$, and $\sqrt[4]{4096}$ is 8 [1]

2.5 A A fractional index means root then power: $16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$. Multiplying 16 by $3/4$ confuses the index with a factor.

3.1 Solve $x^{3/2} = 27$ over the real domain used for this power. Check the result and explain why a negative candidate is invalid. [4]

3.1 Take the nonnegative real square-root convention: $x \geq 0$. $x = 27^{2/3} = 9$; $9^{3/2} = 27$. A negative x has no real square root under this convention.