Course notes

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · SL

AA: course teaching notes

Version: First assessment 2021

These are original teaching notes, and each lesson is labelled with its tier.

1 · Exact arithmetic and estimation

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** 质因数: A prime number that divides the integer exactly.

The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

A worked case

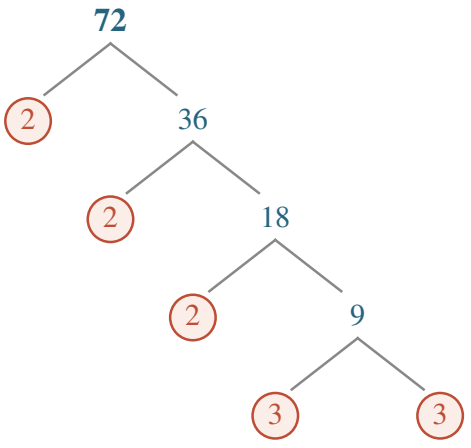
$$a=\prod p_i^{\alpha_i},\quad b=\prod p_i^{\beta_i},\quad \mathrm{HCF}(a,b)=\prod p_i^{\min(\alpha_i,\beta_i)}$$

■ Worked example

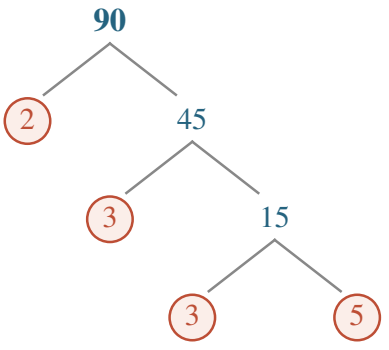
72=2^3×3^2 and 90=2×3^2×5. Their HCF is 2×9=18. Make 18 bags with 4 pencils and 5 pens each. Their LCM is 2^3×3^2×5=360.

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Factor trees: HCF(72, 90) = $2 \times 3^2 = 18$



$72 = 2^3 \times 3^2$



$90 = 2 \times 3^2 \times 5$

Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

■ Watch out

Not true: The HCF of two positive integers is larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

In your course

- Analysis and Approaches SL, the course first assessed in 2021.

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■ **Key idea**

- **Prime factor**: A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

1 · Indices, surds and standard form

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** 指数: The power to which a base is raised.

The method

- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has $1 \leq a < 10$.

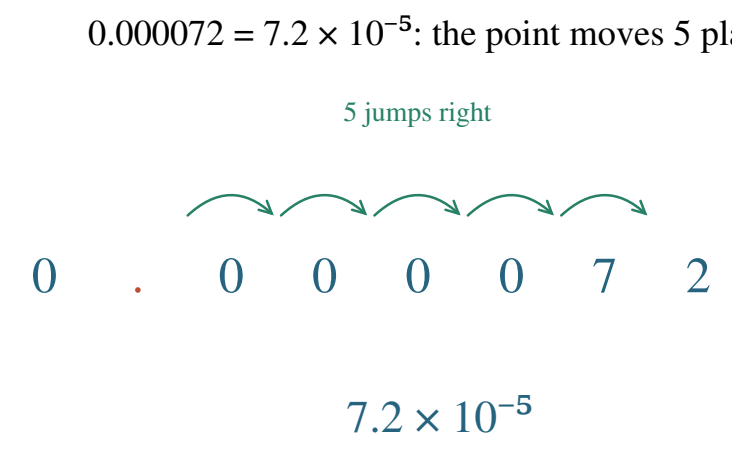
A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

■ **Worked example**

0.000072=7.2×10^(-5). Also 16^(3/4)=(16^(1/4))^3=2^3=8. Simplify √72=6√2, then rationalise 1/√2=√2/2.

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Where it goes wrong

- Index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.
- Do not round a surd when an exact answer is requested.

> **Watch out**
>
> **Not true:** For every positive a, a^2+a^3 equals a^5. Index laws only add powers when the terms are multiplied: $2^2+2^3=12$, but $2^5=32$.

Applying it

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

Key idea

- **Index**: The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: 2^3+2^4=24, not 2^7.

1 · Sequences, series and recurrence

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio** 公比: The constant multiplier between consecutive terms of a geometric sequence.

The method

- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- For a geometric progression, u_n=ar^(n-1) and S_n=a(1-r^n)/(1-r).
- An infinite geometric sum exists only if |r|<1.

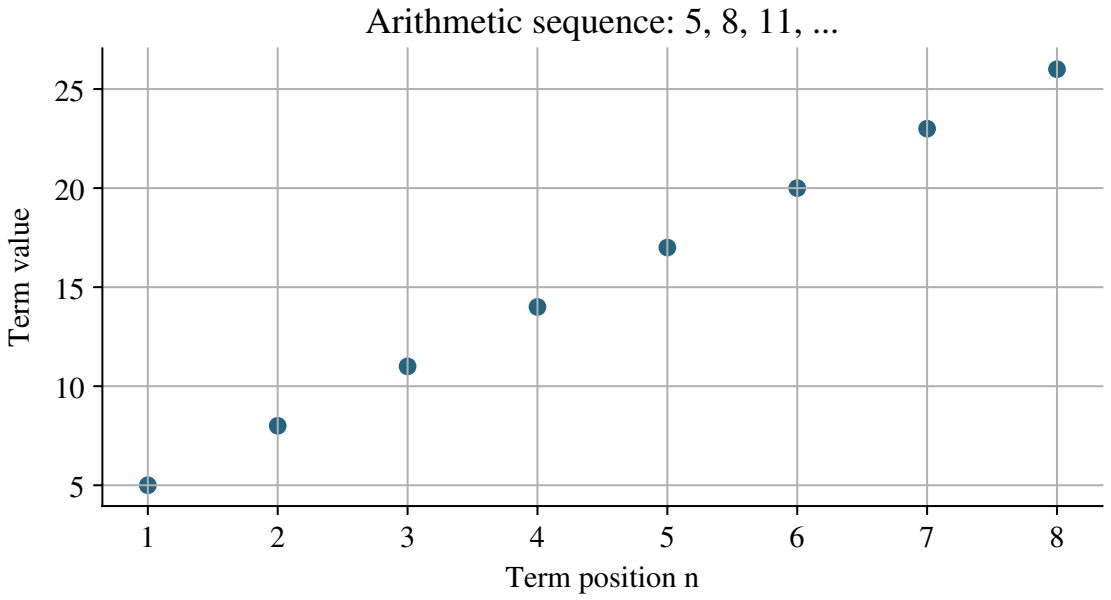
A worked case

$$u_n = a + (n-1)d, \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Worked example

For a=5,d=3,n=8, u_8=5+7×3=26 and S_8=8(10+21)/2=124. For a=12,r=1/2, S infinity=12/(1-1/2)=24. For u_(n+1)=2u_n+1 with u_1=1, the next terms are 3,7,15.

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Where it goes wrong

- The first term has index 1, so the exponent is n-1.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

> ■ **Watch out**
>
> **Not true:** Every geometric series has a finite sum to infinity. A sum to infinity exists only when $-1<r<1$; for $r=2$ the terms grow without limit.

Applying it

- Explain whether the context justifies additive or multiplicative change.
- In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula.

In your course

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■ **Key idea**

- **Common ratio**: The constant multiplier between consecutive terms of a geometric sequence.
- For an arithmetic progression, u_n=a+(n-1)d and S_n=n(2a+(n-1)d)/2.
- Watch for: the first term has index 1, so the exponent is n-1.

1 · Percentages, ratio and proportional reasoning

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier** 乘数: A factor that performs a percentage change in one multiplication.

The method

- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Reverse a percentage by dividing by the multiplier.
- In a ratio, first find the total number of parts.

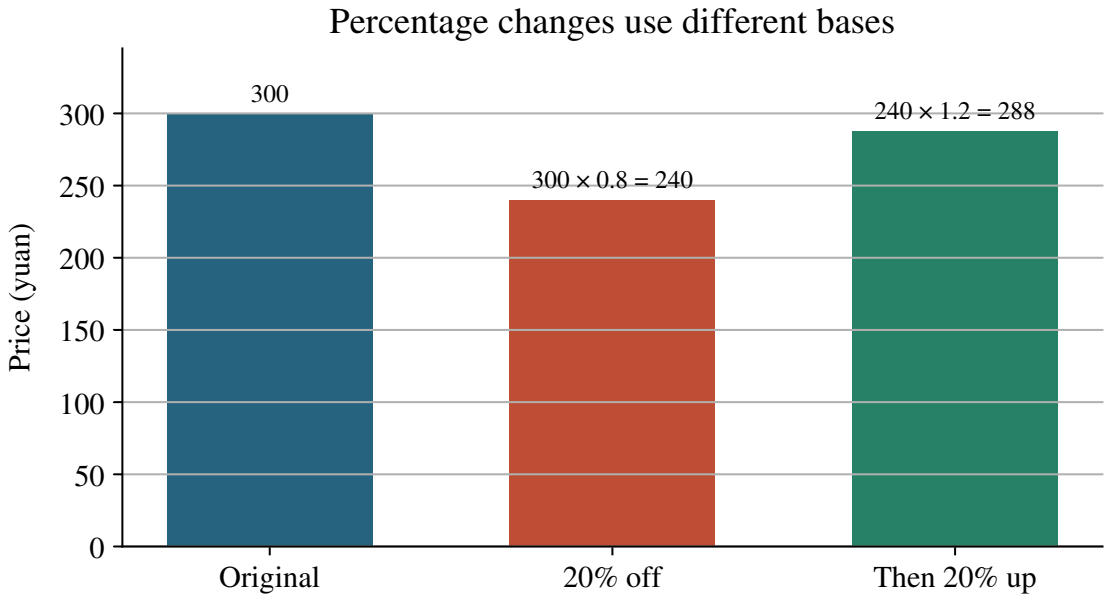
A worked case

$$P_{\text{new}} = P_{\text{old}}\left(1 + \frac{r}{100}\right)$$

■ **Worked example**

Let the original price be P. The model is sale price=0.8P. Hence P=240/0.8=300. A later 20% increase gives 240×1.2=288, so the two changes do not cancel.

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Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

> ■ **Watch out**
>
> **Not true:** A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100\times0.8\times1.2=96$, not 100.

Applying it

- For direct proportion use y=kx; for inverse proportion use y=k/x.
- Calculate k from a known pair before using a new value.
- State what you held constant.

In your course

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■ **Key idea**

- **Multiplier**: A factor that performs a percentage change in one multiplication.
- A p% increase has multiplier 1+p/100; a decrease has multiplier 1-p/100.
- Watch for: A percentage uses a stated base.

2 · Domains, inverses and composition

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** 定义域: The set of allowed inputs to a function.

The method

- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f; the intermediate output must be an allowed input to f.

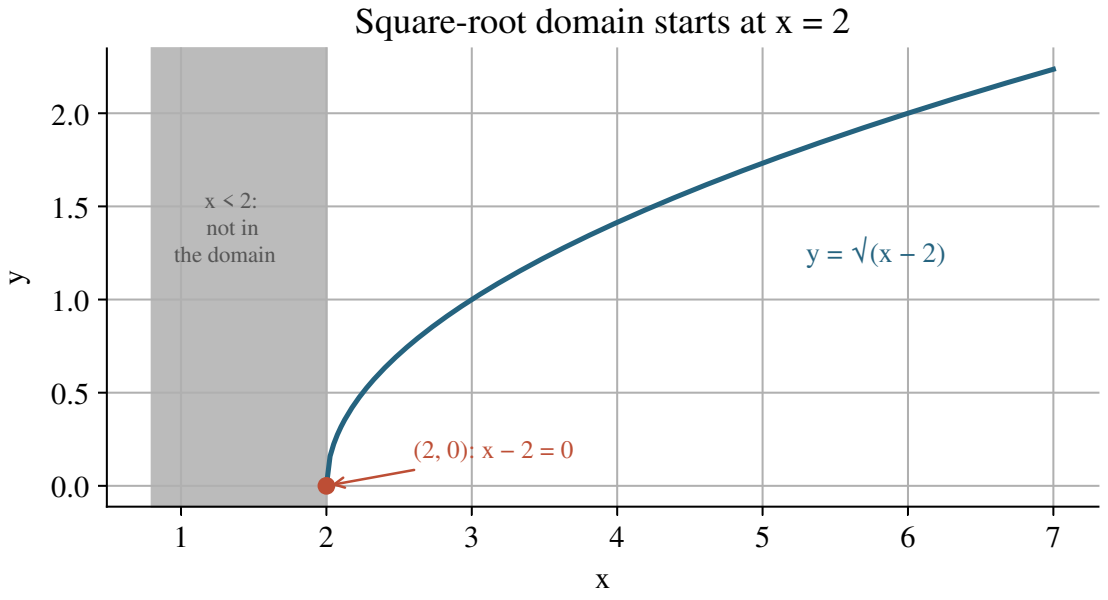
A worked case

$$f(g(x)) = (f \circ g)(x), \qquad f^{-1}(f(x)) = x$$

■ **Worked example**

For f(x)=√(x-2), x≥2 and the range is y≥0. From y=√(x-2), x=y^2+2. Thus f inverse(x)=x^2+2 with x≥0. For g(x)=x+3, fg(1)=f(4)=√2.

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Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

> ■ **Watch out**
>
> **Not true:** Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one (f(1)=f(−1) for x^2), so its domain must be restricted first.

Applying it

- Check f(f inverse(x))=x on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

In your course

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- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

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■ **Key idea**

- **Domain**: The set of allowed inputs to a function.
- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

2 · Exponentials, logarithms and modelling

Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** 半衰期: The time for a decaying quantity to fall to half its initial value.

The method

- For y=Ae^(kt), k is a proportional rate.
- Taking logs gives ln y=ln A+kt.
- Logarithms require positive arguments, and log(x+y) is not log x+log y.

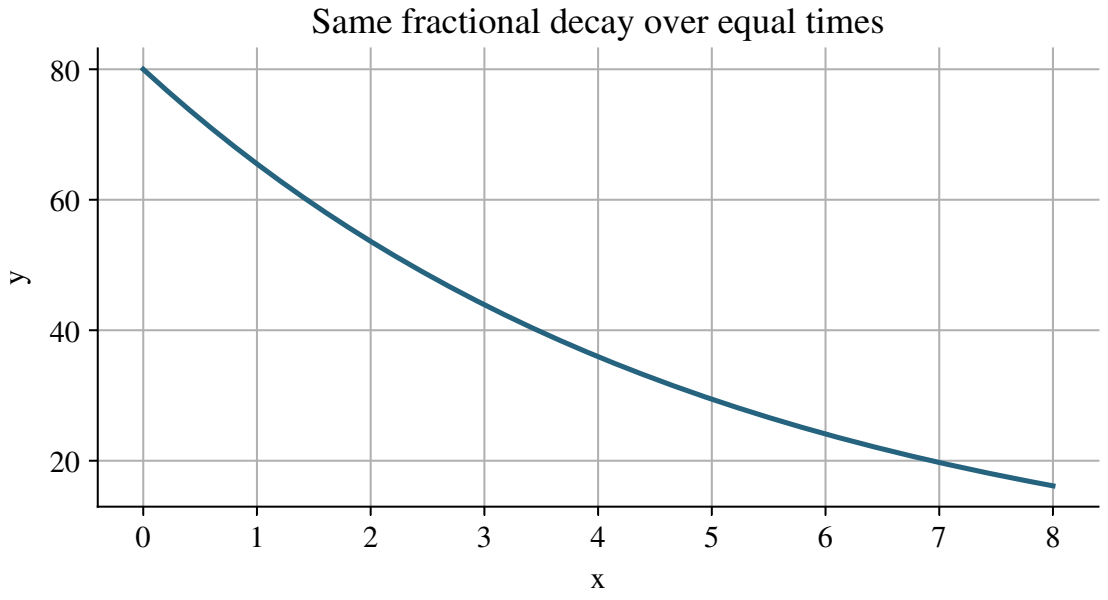
A worked case

$$y = Ae^{kt}, \qquad \ln y = \ln A + kt$$

■ **Worked example**

If y=80e^(-0.2t), y=40 gives e^(-0.2t)=1/2. Hence t=ln2/0.2≈3.466. For 3^x=20, x=ln20/ln3. Plotting ln y against t linearises this exponential model.

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Where it goes wrong

- A fitted exponential is a model, not a guarantee.
- Specify the time units and range of use.
- A negative k describes decay; a negative starting amount usually contradicts the context.

> ■ **Watch out**
>
> **Not true:** For positive x and y, ln(x+y)=ln x+ln y. The log of a sum has no simple law; the product rule is ln(xy)=ln x+ln y.

Applying it

- Compare actual observations with the model.
- Systematic departures may indicate changing conditions.
- In a report, explain what the rate and initial value mean, rather than giving a bare equation.

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In your course

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■ Key idea

- **Half-life**: The time for a decaying quantity to fall to half its initial value.
- For y=Ae^(kt), k is a proportional rate.
- Watch for: A fitted exponential is a model, not a guarantee.

3 · Angle reasoning, similarity and mensuration

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** 相似比: The multiplier that relates corresponding lengths in similar shapes.

The method

- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

A worked case

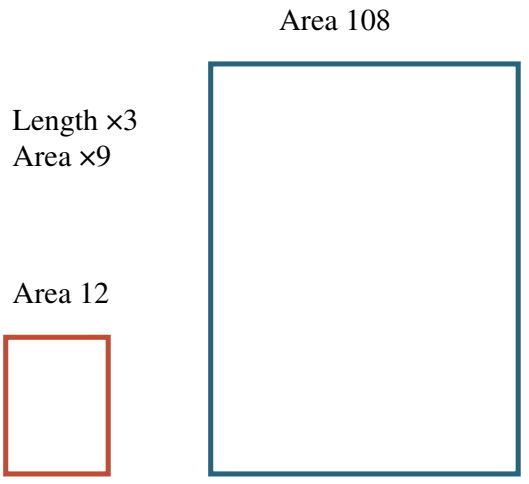
$$\frac{A_2}{A_1} = k^2, \qquad \frac{V_2}{V_1} = k^3$$

■ Worked example

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12\times3^2=108$ cm^2 and a model volume of 8 cm^3 gives $8\times3^3=216$ cm^3. A cylinder with r=3,h=5 has volume $\pi r^2 h=45\pi$.

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A length scale factor is squared for area



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

> ■ **Watch out**
>
> **Not true:** Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

In your course

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■ **Key idea**

- **Scale factor**: The multiplier that relates corresponding lengths in similar shapes.
- For similar shapes with length scale factor k, areas scale by k^2 and volumes by k^3.
- Watch for: equal angles alone establish similarity, not equal size.

3 · Right triangles and non-right triangles

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** 斜边: The side opposite the right angle in a right-angled triangle.

The method

- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- For other triangles, use the sine or cosine rule, or area=ab sin C/2.

A worked case

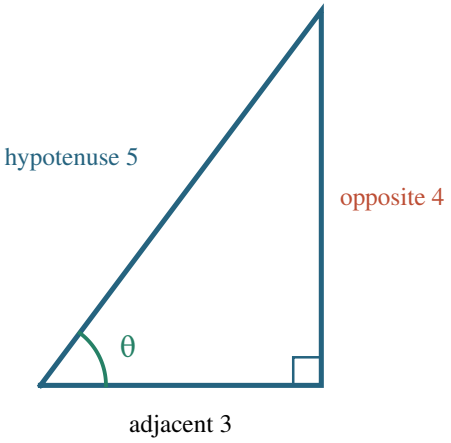
$$a^2 + b^2 = c^2, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ **Worked example**

The ladder length is $c=\sqrt{(3^2+4^2)}=5$ m. Its angle to the ground satisfies $\tan\theta=4/3$, so $\theta\approx53.1°$. With two sides 6 and 8 enclosing 60°, $c^2=6^2+8^2-2\times6\times8\cos60°=52$.

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Label the known sides before choosing a rule



opposite: across from θ
adjacent: next to θ
hypotenuse: across from the right angle

$\tan\theta = 4/3$ $\sin\theta = 4/5$
$\cos\theta = 3/5$

Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

■ Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

In your course

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- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

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■ **Key idea**

- **Hypotenuse**: The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta$=opposite/hypotenuse, $\cos\theta$=adjacent/hypotenuse and $\tan\theta$=opposite/adjacent.
- Watch for: label sides relative to the chosen angle.

3 · Radians, identities and trigonometric equations

How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** 弧度: The angle subtended by an arc whose length equals the radius.

The method

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$.
- Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve.
- List every solution in the stated interval.

A worked case

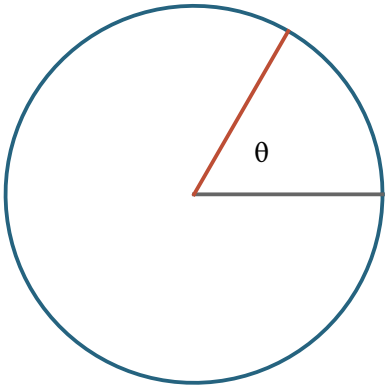
$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta, \qquad \sin^2\theta + \cos^2\theta = 1$$

■ **Worked example**

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\le x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\le 2x<4\pi$ before dividing by 2.

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A radius fixes distance from the centre



radius × angle gives arc length

Where it goes wrong

- A calculator's inverse sine gives one principal value.
- It does not give every solution.
- Dividing by sin x can discard solutions where sin x=0; check that case first.

■ Watch out

Not true: The principal inverse sine value is the only solution in a full turn. Sine takes most values twice in a full turn: sin x=0.5 gives 30° and 150°.

Applying it

- Choose an identity that reduces the equation to one trigonometric function.
- Keep exact familiar angles where possible, and check each solution against the original equation.

In your course

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■ **Key idea**

- **Radian**: The angle subtended by an arc whose length equals the radius.
- For θ in radians, arc length s=rθ and sector area A=r$^2\theta/2$.
- Watch for: A calculator's inverse sine gives one principal value.

4 · Data summaries, histograms and interpretation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** 频率密度: Frequency divided by class width, used as histogram height.

The method

- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

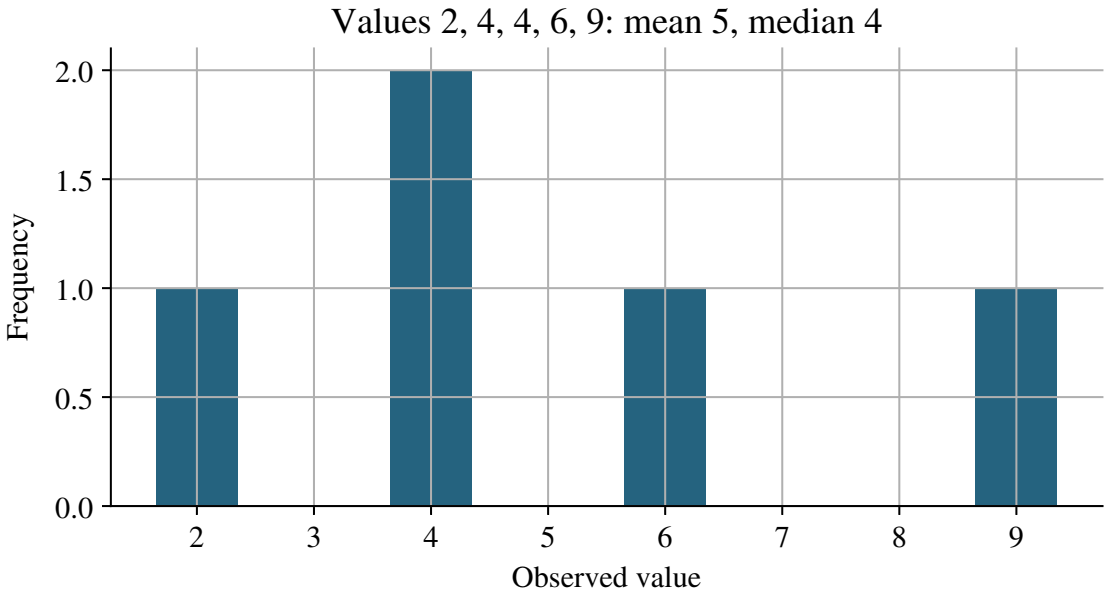
A worked case

$$\overline{x} = \frac{\sum x_i}{n}, \qquad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

■ **Worked example**

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.

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Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

■ Watch out

Not true: A histogram bar's height equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

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In your course

- Analysis and Approaches SL, the course first assessed in 2021.
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■ Key idea

- **Frequency density**: Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

4 · Probability, trees and conditional reasoning

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.

The method

- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

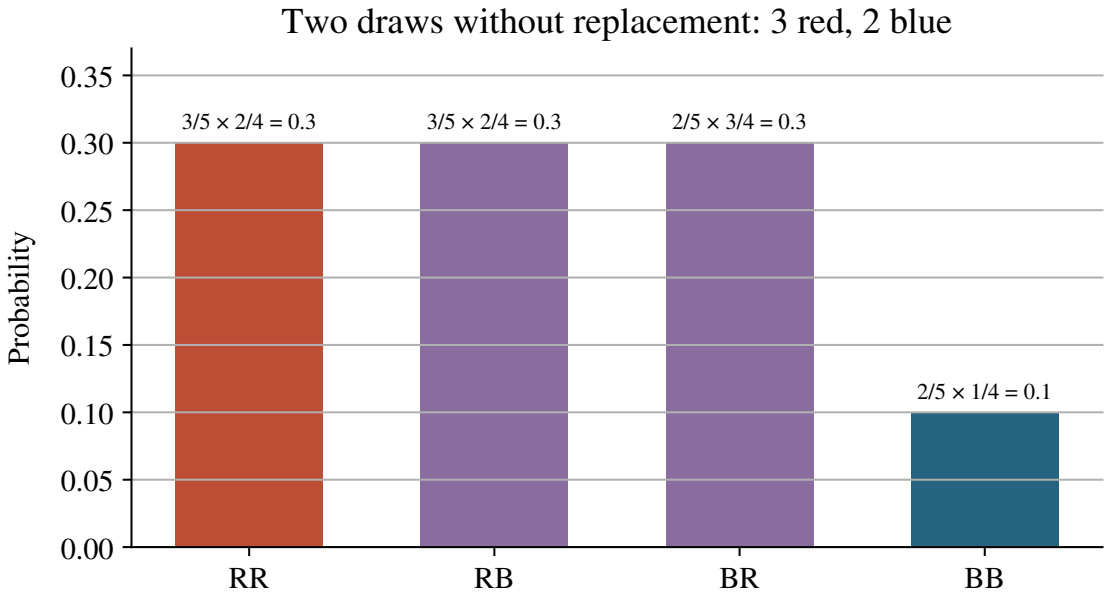
A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

■ Worked example

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/5. If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.

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Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

> ■ **Watch out**
>
> **Not true:** Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B)=0 \neq P(A)P(B)$.

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

In your course

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■ **Key idea**

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

4 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.

The method

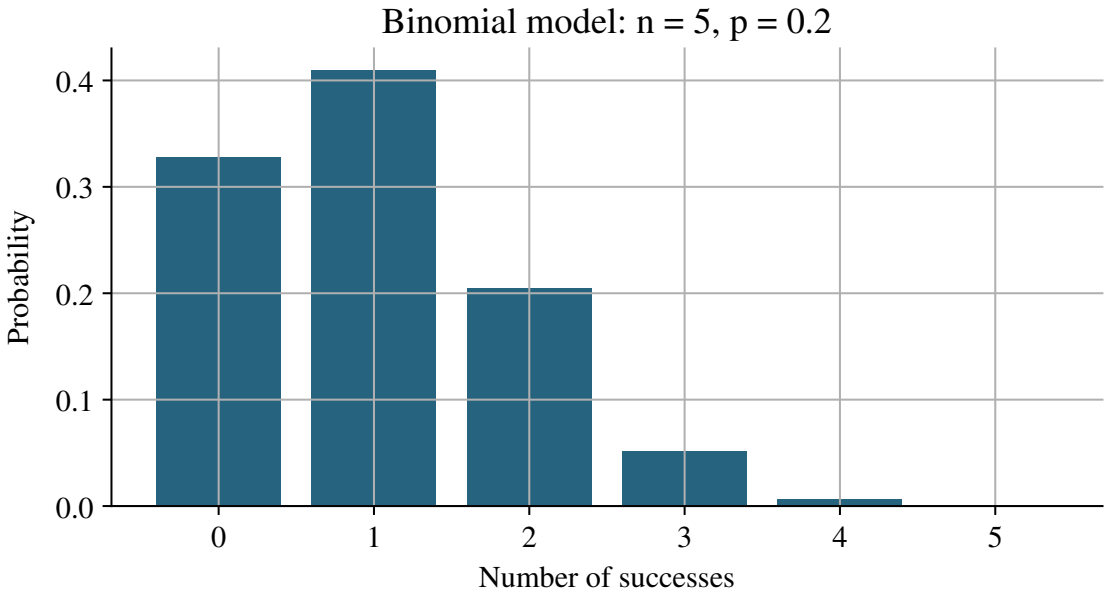
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- E(X)=np and Var(X)=np(1-p).
- For a normal model use z=(x-μ)/σ.
- A Poisson model describes counts with a constant rate and appropriate independence assumptions.

A worked case

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad X \sim B(n, p)$$

■ **Worked example**

For X binomial(5,0.2), P(X=0)=0.8^5=0.32768, E(X)=1 and Var(X)=0.8. For a normal quantity with μ=100,σ=15, the value 130 has z=2. A Poisson mean of 3 per hour gives mean 6 over two hours.



Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

Watch out

Not true: Every count of successes has a binomial distribution regardless of dependence. The binomial model needs independent trials with a fixed probability; dependent trials break it.

Applying it

- Write the event as an inequality before using calculator distribution functions.
- Distinguish $P(X<k)$, $P(X\leq k)$ and a tail complement.
- State assumptions in context.

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In your course

- Analysis and Approaches SL, the course first assessed in 2021.

■ Key idea

- **Expected value**: The probability-weighted mean of a random variable.
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- Watch for: not every count is binomial: changing p or dependence can invalidate it.

5 · Derivatives and stationary points

What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** 导数: The instantaneous rate of change, also the gradient of a tangent.

The method

- For y=ax^n, dy/dx=anx^(n-1).
- A stationary point satisfies dy/dx=0.
- Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.

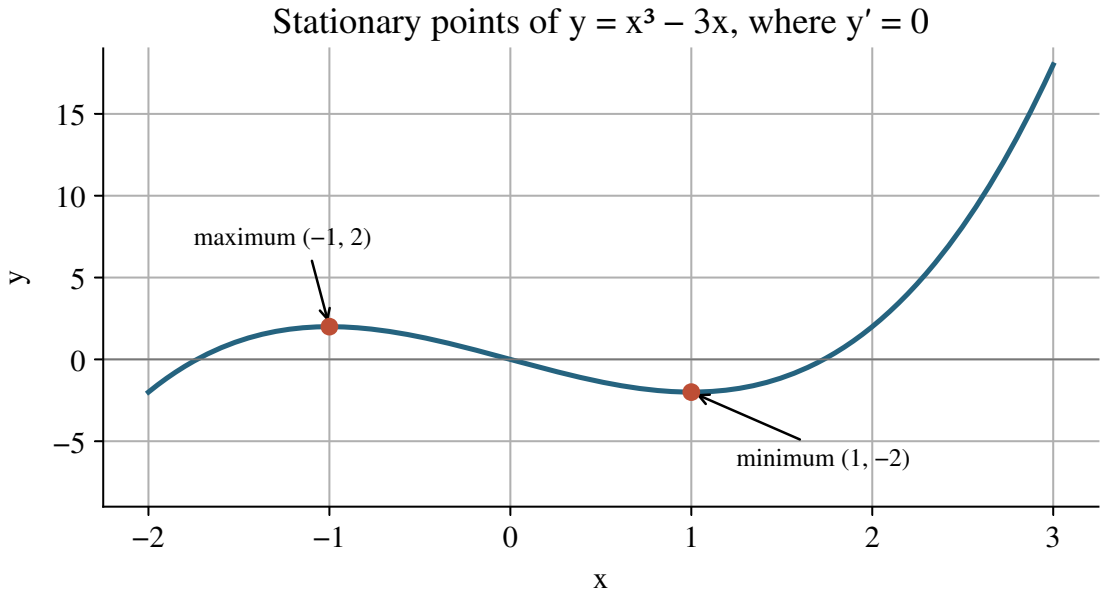
A worked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \qquad f'(x) = 0$$

■ Worked example

For $y=x^3-3x$, $dy/dx=3x^2-3$. At x=1, the gradient is 0 and y=-2. The second derivative is 6x, positive at x=1, so this is a local minimum. At x=-1, y=2 and the second derivative is negative, giving a local maximum.

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Where it goes wrong

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.
- An endpoint can also produce an extreme value on a restricted domain.

> ■ **Watch out**
>
> **Not true:** Every point with zero derivative is a local maximum or minimum. A zero derivative can also be a point of inflection: $y=x^3$ at $x=0$ is neither a maximum nor a minimum.

Applying it

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there.
- At advanced level, connect the derivative to rates and optimization with a valid domain.

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In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Derivative**: The instantaneous rate of change, also the gradient of a tangent.
- For y=ax^n, dy/dx=anx^(n-1).
- Watch for: A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing.

5 · Integrals, area and accumulation

How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** 原函数: A function whose derivative equals the given integrand.

The method

- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- A definite integral is signed accumulation.
- Split at sign changes when total area or distance is required.

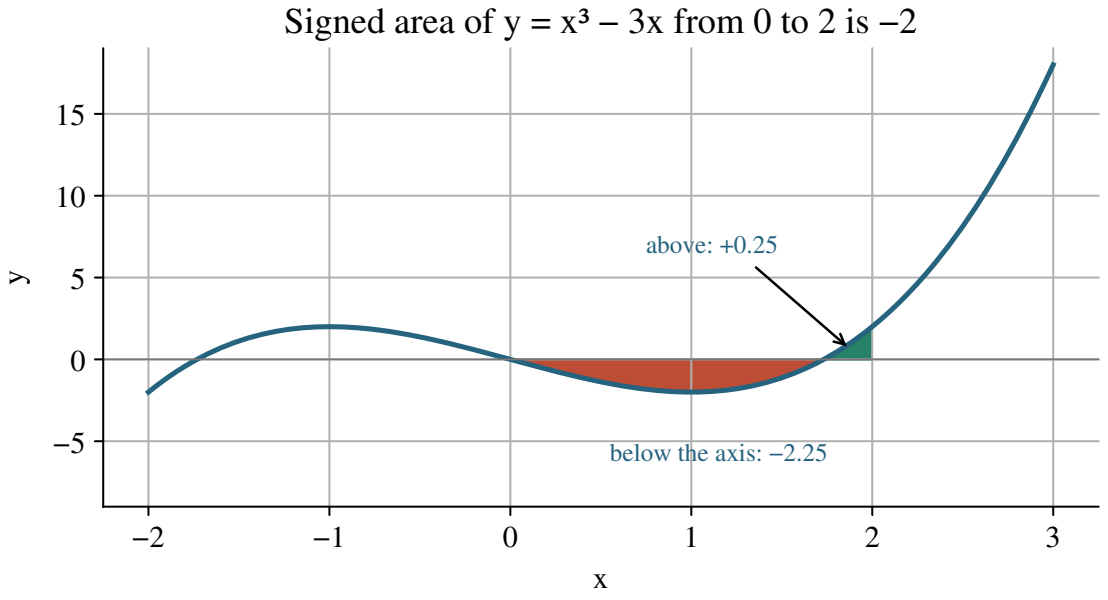
A worked case

$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

■ Worked example

For v(t)=3t^2-3 over 0≤t≤2, displacement=[t^3-3t]_0^2=2. Since v changes sign at t=1, distance=-[t^3-3t]_0^1+[t^3-3t]_1^2=2+4=6. The constants cancel only for a definite integral.

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Where it goes wrong

- The integral of 1/x is ln|x|+C, not the power rule with n=-1.
- Signed area can be zero even when the total enclosed area is positive.

> ■ **Watch out**
>
> **Not true:** A definite integral equals the total positive area, even when the graph crosses the axis. Area below the axis counts as negative, so a definite integral can be smaller than the total area, or even zero.

Applying it

- Identify what the integral means and include the correct units.
- A rate measured per second integrates to the underlying quantity, not to another rate.
- Differentiate an antiderivative to check it.

In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

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■ **Key idea**

- **Antiderivative**: A function whose derivative equals the given integrand.
- For n≠-1, the integral of ax^n is ax^(n+1)/(n+1)+C.
- Watch for: the integral of 1/x is ln|x|+C, not the power rule with n=-1.

IA · Planning a mathematical exploration

What makes an investigation mathematical?

- A student wants to predict queue waiting time at the school canteen. A useful exploration needs a question, data and mathematics the student can explain.
- This lesson studies **assumption** 假设: A condition adopted to make a mathematical model possible and whose effect should be evaluated.

The method

- Frame a focused question with measurable variables and a feasible method.
- Keep a dated record of decisions, sources and mathematical reasoning.
- Use technology to support analysis, and explain every important output in your own words.

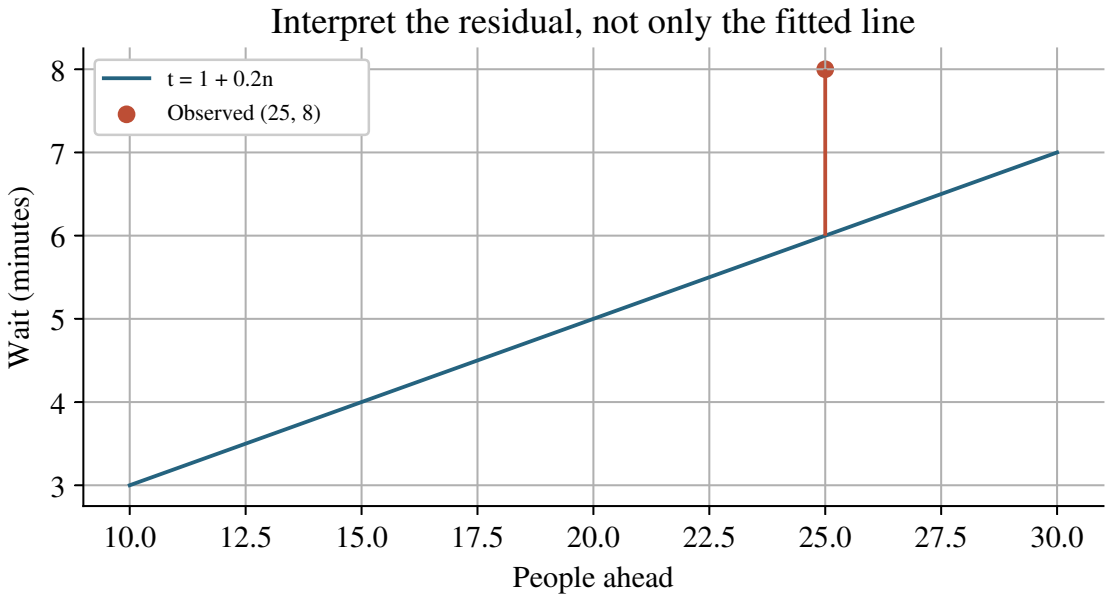
A worked case

$$\widehat{t} = 1 + 0.2n, \qquad e = t - \widehat{t}$$

■ **Worked example**

A pilot records mean waiting times of 3,5,7 minutes when 10,20,30 people are ahead. A first model is t=1+0.2n. At n=25 it predicts 6 minutes. An actual wait of 8 gives residual 2; investigate varying service rates rather than hiding the mismatch.

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Where it goes wrong

- Do not copy a sample exploration, invent data or manufacture personal engagement.
- A prediction is not evidence until it is compared with observations.
- Consent and privacy matter when collecting school data.

■ Watch out

Not true: The school-supervised mathematical exploration can be replaced by a timed written mock exam. The exploration is an extended piece of written work assessed on its own criteria; a timed exam tests something different.

Applying it

- The school supervises topic approval, checkpoints, authenticity and assessment.
- For the current 2021 course use the current school's guide and criteria; the 2029 inquiry criteria belong to a different assessment version.

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In your course

- Analysis and Approaches SL, the course first assessed in 2021.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Assumption**: A condition adopted to make a mathematical model possible and whose effect should be evaluated.
- Frame a focused question with measurable variables and a feasible method.
- Watch for: do not copy a sample exploration, invent data or manufacture personal engagement.

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