The mathematical exploration

Internal assessment · SL and HL

A picture that changed hospitals

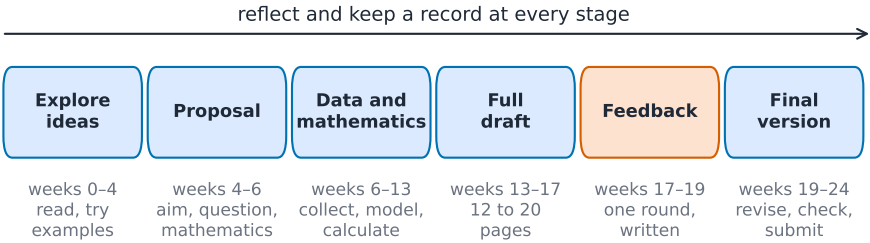
> A real question, mathematics you understand, communicated clearly.

- Nightingale's diagrams of army deaths, 1850s.
- More soldiers died of disease than of wounds.
- This unit: topic $\to$ question $\to$ writing $\to$ modelling $\to$ **reflection**.

The five criteria

Criterion	Marks
A **Presentation**	4
B **Mathematical communication**	4
C **Personal engagement**	3
D Reflection	3
E Use of mathematics	6

Plan the time



- One round of written feedback on a draft.
- Keep dated notes at every stage.

From topic to question

Weak

Maths in music

Basketball

Show Pythagoras' theorem

Better

Do piano frequencies form a geometric sequence?

Which release angle maximises free-throw success?

Extend Pythagoras to a right-angled corner in 3D

Four tests for a topic

interest
you will spend months on it

mathematics
needs course maths, used by you

scope
not "maths in nature", not one calculation

feasibility
you can get the data or do the work

> A good topic passes all four

Correct notation

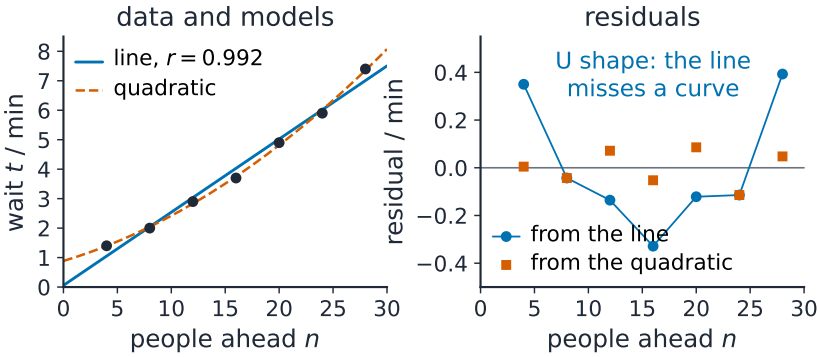
Not this	**This**
0.0012	1.2×10^{-3}
sqrt(x), x*y	$\sqrt{x}$, xy
r^2 = .98	$r^2 = 0.98$, then explain it

Write maths, not calculator

do not write		write
`y = 2.3x^2 + 1.2E-3`	→	$y = 2.3x^2 + 1.2 \times 10^{-3}$
`sqrt(x), x*y, pi`	→	$\sqrt{x},\ xy,\ \pi$
`r^2 = .98`	→	$r^2 = 0.98$, and what it means

- 2.3x^2 becomes $2.3x^2$; 1.2E-3 becomes 1.2×10^{-3}.
- sqrt(x) becomes $\sqrt{x}$.

Residuals reveal the curve



- $t = 0.057 + 0.248n,\ r = 0.992.$
- The U-shaped residuals show a missed curve.

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Why the curve?

1. **Observe.** The popular dish runs out in long queues
2. **Effect.** Servers stop to restock, so service slows
3. **Limit.** One canteen, lunchtime, $4 \leq n \leq 28$

Decision

A better fit needs an explanation

Critical reflection

Descriptive

The model worked well.

I could collect more data.

Critical

Residuals below 0.12 min, about 7 s.

No data beyond $n = 28$, so 10.4 min is unsafe.

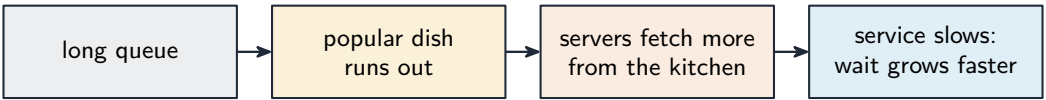
Check: which criterion?

Your turn

A student explains why an assumption failed and how to test a new one. Which criterion does this mainly earn?

A. D Reflection
B. A Presentation
C. B Mathematical communication
D. E Use of mathematics

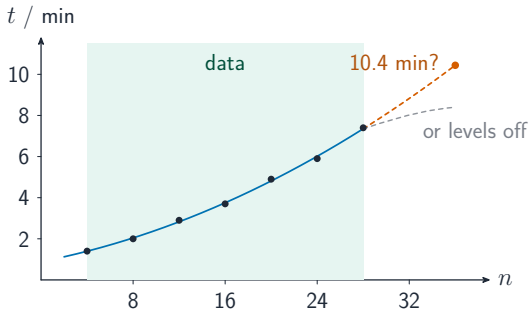
Explain the curve



$t = 0.886 + 0.110n + 0.00432n^2$: residuals below 0.12 min;
the observed reason, not the better fit, makes the curve believable

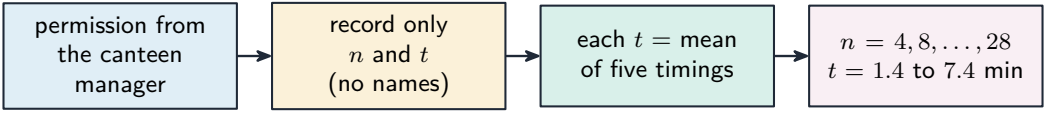
- A long queue empties the popular dish.
- Fetching more slows service, so the wait grows faster.

Extrapolation is a claim



- The quadratic gives 10.4 minutes at $n = 36$.
- A levelling-off model fits the same data.

Planning ethical, reliable data



n from 4 to 28

Integrity

> If you cannot explain a step, it should not be in the exploration.

- Cite data, images, software and AI tools.
- Never invent or alter data.

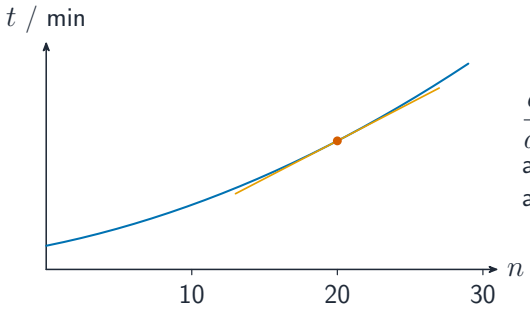
Criterion E at HL

1. **Fit.** $t = 0.886 + 0.110n + 0.00432n^2$
2. **Differentiate.** $\frac{dt}{dn} = 0.110 + 0.00864n$
3. **Interpret.** At $n = 20$, about 17 s per extra person

Decision

Mathematics that answers the question

The rate at a point



$$\frac{dt}{dn} = 0.110 + 0.00864n$$

at $n = 20$: 0.283 min per extra person, and the rate keeps rising

- Tangent at $n = 20$: gradient 0.283 minutes per extra person.
- A derivative turns a model into a rate.

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Exam tips

> Choose a question you care about that needs course mathematics.

- Label every graph; define every variable.
- Look at residuals, not only r.
- Use your one round of feedback fully.

Check yourself

Try these

Rewrite “Maths in football” as a focused research question.

Find the residual for $n = 16$, $t = 3.7$ from $t = 0.057 + 0.248n$.

Turn “my results have human error” into critical reflection.